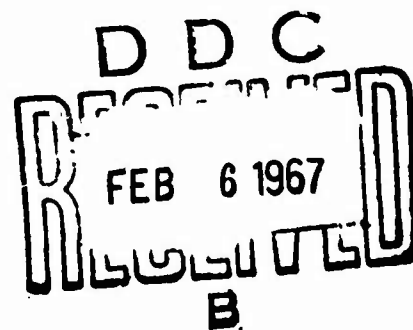
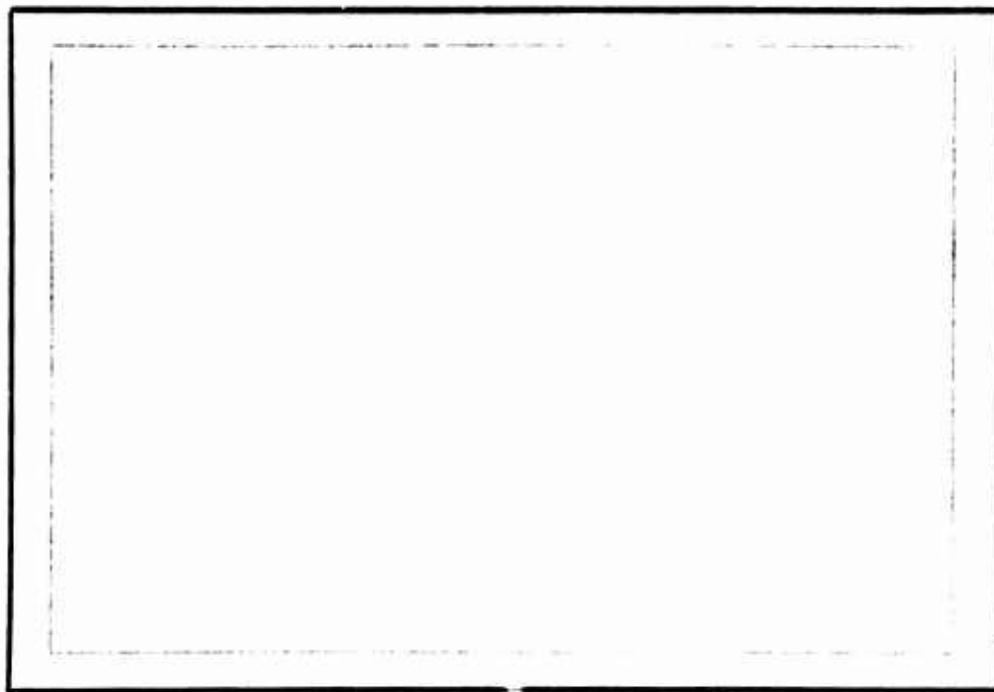


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Report 67-1

Behavior of Stiffened Plates,
Volume I: Analysis

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ABSTRACT

A compilation of techniques of analysis for orthogonally stiffened, flat, rectangular plates, with various combinations of loading and support conditions, is presented. The specific types of plates considered are sandwich, corrugated, rib-reinforced, and integrally stiffened. Orthotropic plate theory is used as the major form of solution throughout the report.

A complete bibliography, consisting of more than three hundred references, is included.

DESCRIPTORS

Corrugations
Integral stiffening
Literature survey
Orthotropy
Plates
Rib-stiffening
Sandwich
Structural analysis

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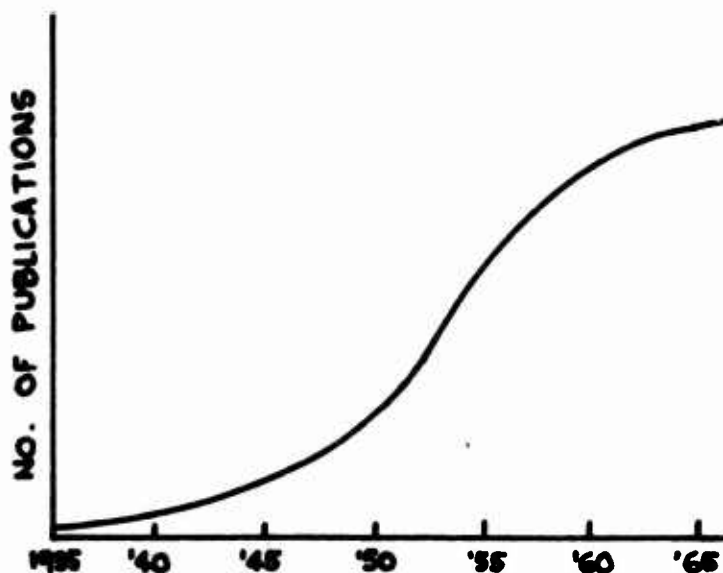
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INTRODUCTION

Historically, the first example of a stiffened plate was a flat slab, reinforced by attached unidirectional girders. Later, stiffening was provided by transverse, as well as longitudinal, beams. Such configurations are still prevalent in the construction of ships and bridges. Concern about saving weight, while retaining strength, in the design of aircraft resulted in the development of sandwich construction, in which two thin plates which carry tensile and compressive loads surround a lightweight, but relatively thick, core designed to transmit shear. Corrugated plates also have been a popular form of construction for many years. However, the most recent developments have been in the area of integrally stiffened plates such as "waffle" and "dimple." The principle upon which all of the latter are based is the removal of material from the neutral axis of a flat plate by rolling, pressing or punching, or upon the reduction of an originally flat thick plate by milling into a thin plate with ribs.

It is interesting to note, from the curve shown below, that interest in orthogonally stiffened plates has been increasing steadily. This, in itself, demonstrates a need for this survey of work in the field.



History of Interest in Stiffened Plates

A great deal of work in the area of structural analysis of orthogonally stiffened plates has been performed during the past twenty years. Many unique techniques of solution have been presented and a great number of specific problems have been solved. But, due to such a wide scattering of work in this area, it has been extremely difficult for the practicing engineer to find, in a reasonable amount of time, solutions to specific cases. For this reason, the writer felt that there was a definite need for consolidation of this research into a single report.

This study, therefore, represents a survey of the most common analytical techniques in the area of

orthotropic plates. Within the body of the paper, methods of solution for sandwich, corrugated, rib-stiffened, and integrally stiffened flat, rectangular plates under static loading conditions are presented. In addition, a complete bibliography is provided for researchers who desire to study specific problems in more detail. Since the titles of most of the listed papers quite satisfactorily explain the contents of the papers, the bibliography should prove to be a useful tool.

Basically, there are three major techniques of analysis of orthogonally stiffened flat plates. The first method is based upon the replacement of the actual plate with a grid system of bars. The major flaw of this method of attack is the neglect of torsional rigidities resulting from interactions of the bars. A second technique is based upon energy concepts. This is rather straight-forward but errors are often introduced due to a failure to satisfy all boundary conditions. The third method of analysis is orthotropic plate theory, which provides continuity of the plate. Although the latter method requires rather difficult solutions for complex boundary conditions, this technique is the most general and most accurate. For this reason, orthotropic plate theory is employed throughout most of this report.

The characteristic equation is first developed and it is then applied to various plate configurations, loading cases, and support conditions. Finally, experimental techniques for determining orthogonality constants are discussed.

The writer wishes to express his gratitude to the U. S. Navy Marine Engineering Laboratory for initially arousing his interest in this area. This report is, to a large extent, based upon work the writer performed for USNMEL during the summers of 1964 and 1965. The writer also wishes to thank Dr. Michael C. Soteriades of the Catholic University of America for his kind advice and to Dr. Richard D. Mathieu of the U. S. Naval Academy for his encouragement during the preparation of this paper. In addition, the writer wishes to thank Mrs. Kathy Jones for typing the manuscript.

PART 1 - DIFFERENTIAL EQUATION OF DEFORMED PLATE

The derivation of the differential equation of the deformed orthotropic plate appears in many of the references (1, 6, 7, 37, 40, 41, 58, 97, etc.) cited here.

In the derivation, it is assumed that: the load acting on the plate is normal to the middle plane; planes normal to middle surface of undeformed plate remain normal after deformation; loads are reacted in shear and flexure; transverse shear deformation can be neglected (this constraint will later be removed in a refinement of the theory).

An element of the plate (Figure 1) is considered. The directions shown are considered positive (right-hand rule is used for moment vectors), and the coordinate axes are chosen to coincide with the principal axes of orthotropy.

1.1 Expressions for Stress and Strain

In order to evaluate the forces and moments of Figure 1, it is first necessary to write the expressions for the strains and the stresses. Letting the subscripts define directions, and using Hooke's Law:

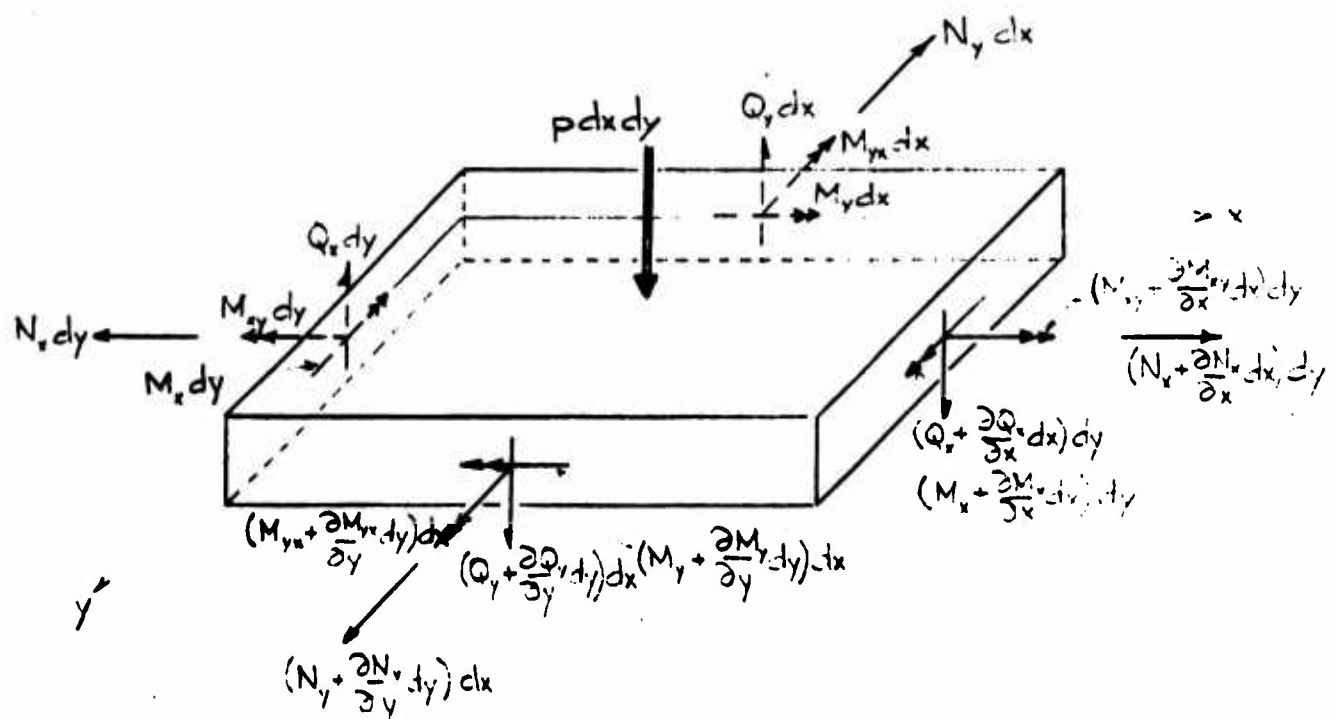


Figure 1
Element $dx-dy$ (positive directions)

$$\left. \begin{aligned} \epsilon_x &= \frac{\sigma_x}{E_x} - \mu_{xy} \frac{\sigma_y}{E_y}, \\ \epsilon_y &= \frac{\sigma_y}{E_y} - \mu_{yx} \frac{\sigma_x}{E_x}, \\ \gamma_{xy} &= \frac{\tau_{xy}}{G_{xy}}, \end{aligned} \right\} (1)$$

where

ϵ, γ = normal and shearing strains, respectively,
 σ, τ = normal and shearing stresses, respectively,
 E, G = moduli of elasticity and rigidity, respectively,
 μ = Poisson's ratio.

Solving these expressions simultaneously gives the stress equations:

$$\left. \begin{aligned} \sigma_x &= \frac{E_x}{1 - \mu_{xy}\mu_{yx}} (\epsilon_x + \mu_{yx}\epsilon_y), \\ \sigma_y &= \frac{E_y}{1 - \mu_{xy}\mu_{yx}} (\epsilon_y + \mu_{xy}\epsilon_x), \\ \tau_{xy} &= \gamma_{xy} G_{xy}. \end{aligned} \right\} (2)$$

It is generally convenient to write the strains in terms of simplified constants with matrix subscripts. Then, the expressions for strains (Eqs. (1)) become:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}, \quad (3)$$

where

$$\begin{aligned}
 S_{11} &= 1/E_x & S_{22} &= 1/E_y \\
 S_{12} &= -\mu_{yx}/E_y & S_{21} &= -\mu_{xy}/E_x \\
 S_{66} &= 1/G_{xy}
 \end{aligned}$$

But, due to symmetry:

$$\frac{\mu_{yx}}{E_x} = \frac{\mu_{xy}}{E_y} \quad \rightarrow \quad S_{12} = S_{21}, \quad (4)$$

and there remain four elastic constants of the orthotropic plate.

1.2 Geometry

Consider a plate element deformed as shown in Figure 2.

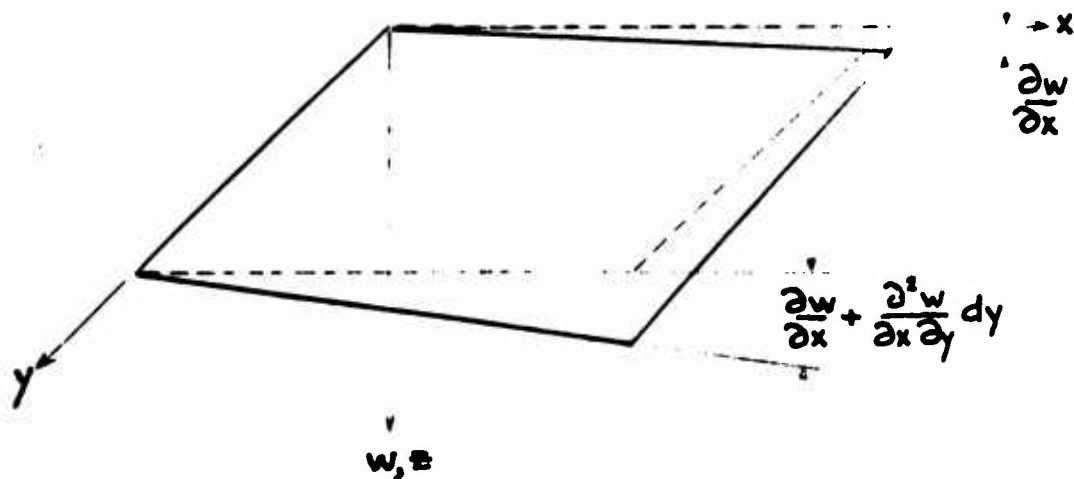


Figure 2
Deformed Element $dx-dy$

The slope of the deformed surface,

$$m_x = \frac{\partial w}{\partial x} \quad (m_y = \frac{\partial w}{\partial y}) \quad (5)$$

The curvature of the deformed surface,

$$\frac{1}{\rho_x} = \frac{\partial^2 w}{\partial x^2} \quad (\frac{1}{\rho_y} = \frac{\partial^2 w}{\partial y^2}), \quad (6)$$

where ρ is the radius of curvature. The "twist" of the surface,

$$\frac{1}{\rho_{xy}} = \frac{\partial^2 w}{\partial x \partial y} \quad (7)$$

If z is the normal distance from the middle surface of the plate to an investigated point, the strains can be written in terms of the geometry:

$$\left. \begin{aligned} \epsilon_x &= -\frac{z}{\rho_x} = -z \frac{\partial^2 w}{\partial x^2}, \\ \epsilon_y &= -\frac{z}{\rho_y} = -z \frac{\partial^2 w}{\partial y^2}, \\ \gamma_{xy} &= \frac{2z}{\rho_{xy}} = 2z \frac{\partial^2 w}{\partial x \partial y}. \end{aligned} \right\} (8)$$

1.3 Final Stress Expressions

The final stresses are evaluated from Eq. (3), by inverting the matrix of elastic constants:

$$\left\{ \begin{aligned} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} &= \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}^{-1} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \\ &= \frac{1}{S_{11}S_{22} - S_{12}^2} \begin{bmatrix} S_{22} & -S_{12} & 0 \\ -S_{12} & S_{11} & 0 \\ 0 & 0 & \frac{S_{11}S_{22} - S_{12}^2}{S_{66}} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \end{aligned} \right\} \quad (9)$$

Substituting the results of Eqs. (8) into Eq. (9):

$$\left\{ \begin{aligned} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} &= \frac{E}{S_{11}S_{22} - S_{12}^2} \begin{bmatrix} S_{22} & -S_{12} & 0 \\ -S_{12} & S_{11} & 0 \\ 0 & 0 & \frac{S_{11}S_{22} - S_{12}^2}{S_{66}} \end{bmatrix} \begin{bmatrix} -\frac{\partial^2 w}{\partial x^2} \\ -\frac{\partial^2 w}{\partial y^2} \\ 2\frac{\partial^2 w}{\partial x \partial y} \end{bmatrix} \end{aligned} \right\} \quad (10)$$

Thus, the stresses,

$$\sigma_x = -\frac{E}{S_{11}S_{22} - S_{12}^2} \left[S_{22} \frac{\partial^2 w}{\partial x^2} - S_{12} \frac{\partial^2 w}{\partial y^2} \right], \quad (11)$$

$$\sigma_y = -\frac{E}{S_{11}S_{22} - S_{12}^2} \left[S_{11} \frac{\partial^2 w}{\partial y^2} - S_{12} \frac{\partial^2 w}{\partial x^2} \right], \quad (12)$$

$$\tau_{xy} = \frac{2E}{S_{66}} \frac{\partial^2 w}{\partial x \partial y}. \quad (13)$$

1.4 Bending and Twisting Moments

Recalling that z is the normal distance from the neutral surface, the stress couples can be evaluated from Eqs. (11), (12), and (13):

$$\begin{aligned} M_x &= \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_x z \, dz \\ &= - \frac{t^3}{12(s_{11}s_{22} - s_{12}^2)} \left[s_{22} \frac{\partial^2 w}{\partial x^2} - s_{12} \frac{\partial^2 w}{\partial y^2} \right], \end{aligned} \quad \left. \vphantom{\int_{-\frac{t}{2}}^{\frac{t}{2}}} \right\} (14)$$

$$\begin{aligned} M_y &= \int_{-\frac{t}{2}}^{\frac{t}{2}} \sigma_y z \, dz \\ &= - \frac{t^3}{12(s_{11}s_{22} - s_{12}^2)} \left[s_{11} \frac{\partial^2 w}{\partial y^2} - s_{12} \frac{\partial^2 w}{\partial x^2} \right], \end{aligned} \quad \left. \vphantom{\int_{-\frac{t}{2}}^{\frac{t}{2}}} \right\} (15)$$

$$\begin{aligned} M_{xy} &= \int_{-\frac{t}{2}}^{\frac{t}{2}} \tau_{xy} z \, dz \\ &= \frac{t^3}{6s_{66}} \frac{\partial^2 w}{\partial x \partial y}. \end{aligned} \quad \left. \vphantom{\int_{-\frac{t}{2}}^{\frac{t}{2}}} \right\} (16)$$

1.5 Rigidity Constants

The bending moment expressions can be rewritten in a simplified form:

$$M_x = - D_x \left[\frac{\partial^2 w}{\partial x^2} + \mu_{xy} \frac{\partial^2 w}{\partial y^2} \right], \quad (17)$$

$$M_y = - D_y \left[\frac{\partial^2 w}{\partial y^2} + \mu_{yx} \frac{\partial^2 w}{\partial x^2} \right], \quad (18)$$

$$M_{xy} = D_r \frac{\partial^2 w}{\partial x \partial y}. \quad (19)$$

where

$$D_x = \frac{E_x t^3}{12(1 - \mu_{xy} \mu_{yx})} \quad (20)$$

= flexural rigidity in x-direction,

$$D_y = \frac{E_y t^3}{12(1 - \mu_{xy} \mu_{yx})} \quad (21)$$

= flexural rigidity in y-direction,

$$D_r = \frac{G_{xy} t^3}{6} \quad (22)$$

= torsional rigidity.

It should be noted, at this point, that various systems for indexing of these rigidity constants are found in the literature. This variety of nomenclatures is extremely confusing due to the fact that a symbol may appear in several technical papers with a different meaning attached to it by each author. Most of the confusion arises from the assignment of symbols to the torsional rigidity constant. The symbol D_R has been chosen here because it does not appear in the reviewed literature, and, it is hoped, will help to avoid misunderstandings.

1.6 Shear Forces

With reference to Figure 1, the equation of equilibrium of forces in the z - direction:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + P = 0. \quad (23)$$

The equation of equilibrium of moments about the x - axis:

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0. \quad (24)$$

From the equilibrium of moments about the y - axis:

$$\frac{\partial M_x}{\partial y} - \frac{\partial M_{xy}}{\partial x} - Q_x = 0, \quad (25)$$

after substitution of

$$M_{xy} = - M_{yx} \quad (26)$$

Substituting Eqs. (17), (18), and (19) into Eqs. (24) and (25), and solving for the shears,

$$Q_x = - D_x \left[\frac{\partial^3 w}{\partial x^3} + \mu_{xy} \frac{\partial^3 w}{\partial x \partial y^2} \right] - D_r \frac{\partial^3 w}{\partial x \partial y^2}, \quad (27)$$

$$Q_y = - D_y \left[\frac{\partial^3 w}{\partial y^3} + \mu_{yx} \frac{\partial^3 w}{\partial x^2 \partial y} \right] - D_r \frac{\partial^3 w}{\partial x^2 \partial y}. \quad (28)$$

1.7 Differential Equation of Flexure

Substitution of Eqs. (24) and (25) makes possible the writing of the equilibrium equation (23) in terms of the moments and the load:

$$\frac{\partial^2 M_x}{\partial x^2} - 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = -p. \quad (29)$$

Finally, from the substitution of Eqs. (17), (18) and (19), the general differential equation of flexure of the orthotropic plate is written:

$$D_x \frac{\partial^4 w}{\partial x^4} + \left[2D_r + \mu_{xy} D_x + \mu_{yx} D_y \right] \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p. \quad (30)$$

For simplicity, the final form of this equation is given as:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = p, \quad (31)$$

where

$$D_{xy} = D_R + \frac{1}{2} (\mu_{xy} D_x + \mu_{yx} D_y). \quad (32)$$

1.8 Consideration of Shear Effects

It is recalled that one of the assumptions of the above derivation was the negligible effect of transverse shear. It must be recognized, however, that such an assumption is not always permissible. In some plates, such as sandwich (see Part 5), a low stiffness gives great importance to the contribution of shear to deflection.

For this reason, several investigators have considered this effect. Reissner (Refs. 78, 169) first recognized this problem, and it has since been applied to orthotropic plates by Crawford and Libove (Ref. 305), Girkman and Beer (Ref. 21), Libove and Batdorf (Ref. 155), March (Ref. 159), Medwadowski (Refs. 62, 63), Suchar (Ref. 93), and Wang (Ref. 177), among others.

Using matrix notation consistent with that of Eq. (3), the stress-strain relationship is given by:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{21} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{31} & s_{32} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix} \quad (33)$$

Due to symmetry,

$$s_{12} = s_{21}, \quad s_{23} = s_{32}, \quad s_{13} = s_{31},$$

and the elastic properties of the plate are characterized by nine elastic material constants. Thus, the effect of transverse shear, together with that of transverse normal stress, is considered.

Medwadowski (Ref. 62) applies nonlinear theory of elasticity to the derivation of the characteristic equation of the orthotropic plate. He formulates the problem by using a system of sixteen simultaneous equations -- Eq. (33), equilibrium equations, and strain-displacement relationships. The problem is reduced to a two-dimensional one by the introduction of body-force resultants, X'' and Y'' , and weighted displacements, u , v , and w . With the introduction of the Airy stress function, Φ , Medwadowski reduces the

number of equations to the following four:

$$\left. \begin{aligned} \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + P_1 - P_2 + Z' - X' \frac{\partial w}{\partial x} - Y' \frac{\partial w}{\partial y} + Q \\ = 0, \end{aligned} \right\} (34)$$

$$\left. \begin{aligned} Q_x + A_1 \frac{\partial^2 Q_x}{\partial x^2} + A_2 \frac{\partial^2 Q_x}{\partial y^2} = A_3 \frac{\partial^3 w}{\partial x^3} + A_4 \frac{\partial^3 w}{\partial x \partial y^2} \\ + A_9 \left(\frac{\partial P_1}{\partial x} - \frac{\partial P_2}{\partial x} \right) + A_{10} \frac{\partial Z'}{\partial x} + X'' + A_{34} \frac{\partial}{\partial x} \left(X' \frac{\partial w}{\partial x} + Y' \frac{\partial w}{\partial y} - Q \right), \end{aligned} \right\} (35)$$

$$\left. \begin{aligned} Q_y + A_5 \frac{\partial^2 Q_y}{\partial x^2} + A_6 \frac{\partial^2 Q_y}{\partial y^2} = A_7 \frac{\partial^3 w}{\partial x^2 \partial y} + A_8 \frac{\partial^3 w}{\partial y^3} \\ + A_{11} \left(\frac{\partial P_1}{\partial y} - \frac{\partial P_2}{\partial y} \right) + A_{12} \frac{\partial Z'}{\partial y} + Y'' + A_{35} \frac{\partial}{\partial y} \left(X' \frac{\partial w}{\partial x} + Y' \frac{\partial w}{\partial y} - Q \right), \end{aligned} \right\} (36)$$

$$\left. \begin{aligned} S_{22} \frac{\partial^4 \Phi}{\partial x^4} + (2S_{12} + S_{66}) \frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + S_{11} \frac{\partial^4 \Phi}{\partial y^4} \\ = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - (S_{12} + S_{22}) \frac{\partial^2 P}{\partial x^2} - (S_{11} + S_{12}) \frac{\partial^2 P}{\partial y^2}, \end{aligned} \right\} (37)$$

where

A_j = constant, depending on elastic properties and thickness of plate,

P_1, P_2 = loads acting on upper and lower faces of plate, respectively,

P = resultant body-force potential,

X', Y', Z', X'', Y'' = body-force resultants,

Φ = Airy stress functions,

$$Q = Q_x \frac{\partial^2 w}{\partial x^2} + 2Q_{xy} \frac{\partial^2 w}{\partial x \partial y} + Q_y \frac{\partial^2 w}{\partial y^2},$$

and the remaining symbols have their normal meanings, as used throughout this paper.

The effect of an elastic foundation at face $z = t/2$ is considered, and a system of four partial differential equations, governing the oscillations of an orthotropic plate, is written. The system is then linearized and reduced to a single partial differential equation of sixth order through the choice of

$$F(x, y, z) = \text{stress function.}$$

The values of the constants contained in this fundamental expression are then given, and the solution of the equation is taken to be of the Lévy-type.

When the body-force terms and the effect of transverse normal stress are neglected and elastic foundation modulus is set equal to zero, Medwadowski's linearized equations which govern the bending behavior reduce to the less complex expressions of Libove and Batdorf (Ref. 155).

Since the Libove-Batdorf paper uses a definition of plate rigidities which is somewhat unique, no attempt is made here to make these conform to the definitions used in the previous derivations. Therefore, let

$$\left. \begin{aligned} D_1 &= -\frac{M_x}{\partial^2 w / \partial x^2} , \\ D_2 &= -\frac{M_y}{\partial^2 w / \partial y^2} , \end{aligned} \right\} (38)$$

analogous to the constants defined by Eqs. (17) and (18).

The twisting rigidity retains its definition as given by Eq. (19):

$$D_R = \frac{M_{xy}}{\partial^2 w / \partial x \partial y} .$$

In addition, Poisson's ratios are defined in terms of curvatures:

$$\left. \begin{aligned} \nu_{yx} &= -\frac{\partial^2 w / \partial y^2}{\partial^2 w / \partial x^2} , \\ \nu_{xy} &= -\frac{\partial^2 w / \partial x^2}{\partial^2 w / \partial y^2} . \end{aligned} \right\} (39)$$

The refinement to the orthotropic plate theory, which was presented in the first part of this section, is introduced in the form of shear stiffnesses,

$$D_{Q_x} = \frac{Q_x}{\partial w / \partial x} , \quad \downarrow (40)$$

$$D_{Q_y} = \frac{Q_y}{\partial w / \partial y} .$$

} (40)

The bending moment equations which originally contained flexure terms only (Eqs. (17), (18), and (19)) are now refined by the consideration of the shear contribution:

$$M_x = \frac{D_1}{1 - \nu_{yx} \nu_{xy}} \left[\frac{\partial^2 w}{\partial x^2} - \frac{1}{D_{Q_x}} \frac{\partial Q_x}{\partial x} + \nu_{xy} \left(\frac{\partial^2 w}{\partial y^2} - \frac{1}{D_{Q_y}} \frac{\partial Q_y}{\partial y} \right) \right], \quad (41)$$

$$M_y = \frac{D_2}{1 - \nu_{yx} \nu_{xy}} \left[\frac{\partial^2 w}{\partial y^2} - \frac{1}{D_{Q_y}} \frac{\partial Q_y}{\partial y} + \nu_{yx} \left(\frac{\partial^2 w}{\partial x^2} - \frac{1}{D_{Q_x}} \frac{\partial Q_x}{\partial x} \right) \right], \quad (42)$$

$$M_{xy} = D_3 \left[\frac{\partial^2 w}{\partial x \partial y} - \frac{1}{2 D_{Q_x}} \frac{\partial Q_x}{\partial y} \right]. \quad (43)$$

Substituting the above equations into Eqs. (24) and (25), and applying the equation of vertical equilibrium:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = -P + N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2}, \quad (44)$$

three equations in w , Q_x , and Q_y are written:

$$\left\{ \begin{aligned} & \left[N_x \frac{\partial^2}{\partial x^2} + 2N_{xy} \frac{\partial^2}{\partial x \partial y} + N_y \frac{\partial^2}{\partial y^2} \right] w + \left[\frac{\partial}{\partial x} \right] Q_x \\ & + \left[\frac{\partial}{\partial y} \right] Q_y = -p, \end{aligned} \right\} \quad (45)$$

$$\left\{ \begin{aligned} & \left[-D_R \frac{\partial^3}{\partial x^2 \partial y^2} - \frac{D_1}{1-v_{xy}v_{yx}} \left(v_{xy} \frac{\partial^3}{\partial x \partial y^2} + \frac{\partial^3}{\partial x^3} \right) \right] w \\ & + \left[\frac{1}{2} \frac{D_R}{D_{Q_x}} \frac{\partial^2}{\partial y^2} + \frac{D_1}{(1-v_{xy}v_{yx})D_{Q_x}} \frac{\partial^2}{\partial x^2} - 1 \right] Q_x \\ & + \left[\frac{1}{2} \frac{D_R}{D_{Q_y}} \frac{\partial^2}{\partial x \partial y} + \frac{D_1 v_{xy}}{(1-v_{xy}v_{yx})D_{Q_y}} \frac{\partial^2}{\partial x \partial y} \right] Q_y = 0, \end{aligned} \right\} \quad (46)$$

$$\left\{ \begin{aligned} & \left[-D_R \frac{\partial^3}{\partial x^2 \partial y} - \frac{D_3}{1-v_{xy}v_{yx}} \left(v_{yx} \frac{\partial^3}{\partial x^2 \partial y} + \frac{\partial^3}{\partial y^3} \right) \right] w \\ & + \left[\frac{1}{2} \frac{D_R}{D_{Q_x}} \frac{\partial^2}{\partial x \partial y} + \frac{D_3 v_{yx}}{(1-v_{xy}v_{yx})D_{Q_x}} \frac{\partial^2}{\partial x \partial y} \right] Q_x \\ & + \left[\frac{1}{2} \frac{D_R}{D_{Q_y}} \frac{\partial^2}{\partial x^2} + \frac{D_3}{(1-v_{xy}v_{yx})D_{Q_y}} \frac{\partial^2}{\partial y^2} - 1 \right] Q_y = 0. \end{aligned} \right\} \quad (47)$$

These equations are then solved for w , Q_x , and Q_y by means of determinants.

1.9 Summary

In general, orthotropic plate theory which omits the effects of transverse shear gives good results for orthotropic or orthogonally stiffened flat plates. Due to the complexity of the problem, it is often worthwhile to "trade off" a small degree of accuracy for a great amount of computational labor. However, the analyst must take care that these errors do not become significant.

Summarizing the results of the bending theory of orthotropic plates, the differential equation of flexure is recalled:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P,$$

where

$$\begin{aligned} D_x &= \frac{E_x t^3}{12(1 - \mu_{xy}\mu_{yx})}, \\ D_y &= \frac{E_y t^3}{12(1 - \mu_{xy}\mu_{yx})}, \\ D_{xy} &= \frac{G_{xy} t^3}{6} + \frac{1}{2} (\mu_{xy} D_x + \mu_{yx} D_y). \end{aligned}$$

The stresses, as given by Eqs. (11), (12), and (13),

$$\sigma_x = - \frac{z}{S_{11}S_{22} - S_{12}^2} \left[S_{22} \frac{\partial^2 w}{\partial x^2} - S_{12} \frac{\partial^2 w}{\partial y^2} \right] ,$$

$$\sigma_y = - \frac{z}{S_{11}S_{22} - S_{12}^2} \left[S_{11} \frac{\partial^2 w}{\partial y^2} - S_{12} \frac{\partial^2 w}{\partial x^2} \right] ,$$

$$\tau_{xy} = \frac{2z}{S_{66}} \frac{\partial^2 w}{\partial x \partial y} ,$$

where

$$S_{11} = \frac{1}{E_x} , \quad S_{22} = \frac{1}{E_y} ,$$

$$S_{12} = - \frac{\mu_{xy}}{E_y} = - \frac{\mu_{yx}}{E_x} , \quad S_{66} = \frac{1}{G_{xy}} .$$

The practical application of orthotropic plate theory depends upon the solution of the lateral deflection, w . This function is found by solving the fundamental equation (31) for a given set of boundary conditions. Then, it is possible to evaluate stresses, strains, moments, and shears. The application of the theory to specific cases of bending is considered in the following section.

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PART 2 - METHODS OF SOLUTION - TRANSVERSE LOADING

The solution of the basic differential equation (31) of the orthotropic plate,

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P ,$$

is accomplished by one of the following methods:

2.1 Navier Solution

A double Fourier series solution of the form

$$f(x,y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} , \quad (48)$$

was used by a majority of investigators. Huber (Refs. 37 - 41) was the first to give his attention to orthogonally reinforced plates and his approach was based on the Navier-type function. Some of the other investigators to employ this method of attack were Csonka (Ref. 220), Giencke (Ref. 239), Heller (Ref. 309), Kaczowski (Refs. 48, 49), Nowacki (Ref. 71), Raville (Ref. 168), Robinson (Ref. 79), Schumann (Ref. 278), Soper (Ref. 90), Timoshenko and Woinowsky-Krieger (Ref. 79), and Tolotti and Grioli (Ref. 290).

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The Navier solution is simple and straightforward, even for complex loading conditions. However, the method becomes cumbersome in application due to its relatively slow convergence property. It is most undesirable, from a computational standpoint, when higher derivatives of the deformation function, w , are involved.

2.2 Lévy Solution

The Lévy-type solution is based on a single series expression of the form:

$$w = \sum_{m=1}^{\infty} Y_m \sin \frac{m\pi x}{a}, \quad (49)$$

where Y_m is dependent upon y and independent of x . This technique, which is generally more efficient than the above type, was employed by several investigators, among them Ando (Ref. 209), Cornelius (Ref. 12), Hajek (Ref. 25), Lekhnitsky (Ref. 58), Schade (Refs. 273-276), and Timoshenko and Woinowsky-Krieger (Ref. 97).

2.3 Other Solutions

Most of the recent work has been concerned with improving upon the computational labor involved in

solving the fundamental equation. Solutions have been submitted in many forms:

2.3.1 Maclaurin's Series

Rajappa and Reddy (Ref. 77) have applied Maclaurin's series to the problem of the simply supported rectangular plate and written the deformation equation:

$$w(x,y) = w(0,0) + \sum_n \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} \right)^n \frac{w(0,0)}{n!} . \quad (50)$$

with the origin of the coordinate system taken at the center of the plate.

2.3.2 Affine Transformation

Brilla (Ref. 8) makes use of the transformation,

$$x = x' , \quad y = y' \sqrt{\frac{D_y}{D_x}} , \quad (51)$$

to solve this problem which satisfies the condition:

$$D_{xy} = \sqrt{D_x D_y} . \quad (52)$$

The condition of Eq. (52) was recommended by Huber (Ref. 40) for reinforced concrete slabs, and is discussed by Timoshenko and Woinowsky-Krieger (Ref. 97).

2.3.3 Perturbation Technique

Vinson with Brull and Hess (Refs. 34, 101-103) make use of the same transformation as Brilla (Eq. (41)), and then go on to obtain a perturbation expansion in terms of α for plates with rigidity ratios which satisfy the condition,

$$\frac{D_{xy}}{D_x} = \sqrt{\frac{D_y}{D_x}} \quad (53)$$

The parameter, α , measures the deviation from this condition and is given by:

$$\alpha = 2 \left(1 - \frac{D_{xy}}{\sqrt{D_x D_y}} \right) \quad (54)$$

This expression, together with the transformation, changes the basic differential equation (31) to:

$$\nabla^4 w - \alpha \frac{\partial^4 w}{\partial \bar{x}^2 \partial \bar{y}^2} = \frac{p}{D_x} \quad (55)$$

A comparison between this method and the standard techniques is made in Reference 101. It is shown that for the case of a uniformly loaded rectangular orthotropic plate on simple supports, the perturbation solution gives results of reasonable accuracy, while requiring considerably less computation time than the Navier and Lévy techniques.

2.3.4 Complex Variables

Mader (Ref. 61) considers the special condition specified by Eq. (52). Here, the basic differential

equation (31) is written:

$$D_x - 2D_{xy} m^2 + D_y m^4 = 0, \quad (56)$$

where

$$m = \pm \sqrt{\frac{D_{xy}}{D_y} \left[1 \pm \sqrt{1 - \frac{D_x D_y}{D_{xy}^2}} \right]}.$$

The solution has the form:

$$w = \left[C_1 e^{\alpha(\theta+i\phi)y} + C_2 e^{-\alpha(\theta-i\phi)y} + C_3 e^{-\alpha(\theta+i\phi)y} + C_4 e^{-\alpha(\theta-i\phi)y} \right] \sin \alpha x, \quad (57)$$

where

$$\alpha = a \sqrt{\frac{D_x}{D_y}},$$

$$\beta = \pm \alpha (\theta \pm i\phi) = ma.$$

Suchar (Ref. 93) introduces the complex variables,

$$\bar{z} = x + iy \quad \bar{\bar{z}} = x - iy, \quad (58)$$

and parameters suggested by Lekhnicky (Ref. 58),

$$k_1 = \frac{1+i\mu_1}{1-i\mu_1}, \quad k_2 = \frac{1+i\mu_2}{1-i\mu_2}, \quad (59)$$

to rewrite the basic differential equation of the orthotropic plate:

$$\left[k_1 \frac{\partial^2 W}{\partial \bar{z}^2} - (1-k_1 \bar{k}_1) \frac{\partial^2 W}{\partial \bar{z} \partial \bar{z}} + \bar{k}_1 \frac{\partial^2 W}{\partial \bar{z}^2} \right] \left[k_2 \frac{\partial^2 W}{\partial \bar{z}^2} - (1+k_2 \bar{k}_2) \frac{\partial^2 W}{\partial \bar{z} \partial \bar{z}} + \bar{k}_2 \frac{\partial^2 W}{\partial \bar{z}^2} \right] = 0, \quad (60)$$

in which

$$w(x, y) = W(\bar{z}, \bar{z})$$

is a real solution of the characteristic equation.

In addition, Morkovin (Ref. 65) also employs a complex variable approach to the problem.

2.3.5 Energy Technique

A direct method of calculating stresses in orthotropic plates, without the usual intermediate step of calculating deflections, is presented by Coull (Ref. 13). The method of least work is employed.

The total strain energy due to bending is given as:

$$U = \frac{6}{t^3} \int_0^{2b} \int_{-c}^c \left[A_{11} M_x^2 + A_{22} M_y^2 + A_{33} M_{xy}^2 + 2A_{12} M_x M_y + \frac{t^2}{10} B_{11} (N_x^2 + N_y^2) \right] dy dx, \quad (61)$$

where the constants,

$$\begin{aligned} A_{11} &= \frac{E_y}{E_x E_y - E_{xy}^2} & A_{22} &= \frac{E_x}{E_x E_y - E_{xy}^2} \\ A_{12} &= -\frac{E_{xy}}{E_x E_y - E_{xy}^2} & A_{33} &= \frac{1}{G_{xy}} \\ B_{11} &= \frac{1}{G_x} \end{aligned}$$

The plate considered is supported along, and parallel to, the y-axis, and it is free along the two remaining edges.

The moments and forces in the plate are expressed in terms of a single series, and two loading cases, symmetrical and antisymmetrical, are utilized. The solution involves an assumption of an n -th order polynomial for the bending moment M_x , which makes possible the writing of a set of n linear differential equations with constant coefficients.

2.3.6 Design Application

An interesting and useful set of tables for the design of orthotropic plates with various edge supports and under various loadings is given in a dual-language (English and German) book by Krug and Stein (Ref. 57).

The basic equation (31) is used,

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P,$$

and the following notations and transformations are introduced:

$$x = \frac{D_{xy}}{\sqrt{D_x D_y}} \quad \lambda = \frac{D_x}{D_y} \quad (62)$$

$$\xi = \frac{x}{L} \quad \eta = \frac{y}{L} \sqrt{\lambda} \quad (63)$$

$$L_\xi = \frac{L_x}{L} \quad L_\eta = \frac{L_y}{L} \sqrt{\lambda} \quad (64)$$

Using the above notation, the differential equation of the unloaded plate becomes

$$\frac{\partial^4 w}{\partial \xi^4} + 2x \frac{\partial^4 w}{\partial \xi^2 \partial \eta^2} + \frac{\partial^4 w}{\partial \eta^4} = 0. \quad (65)$$

In order to reduce the number of parameters necessary to define a particular plate from three to two (x and e), the loading is referred to the ξ, η -coordinate system (Fig. 3) and the ratio,

$$e = \frac{L_y}{L_x} \sqrt{\lambda} = \frac{L_\eta}{L_\xi}, \quad (66)$$

is introduced.

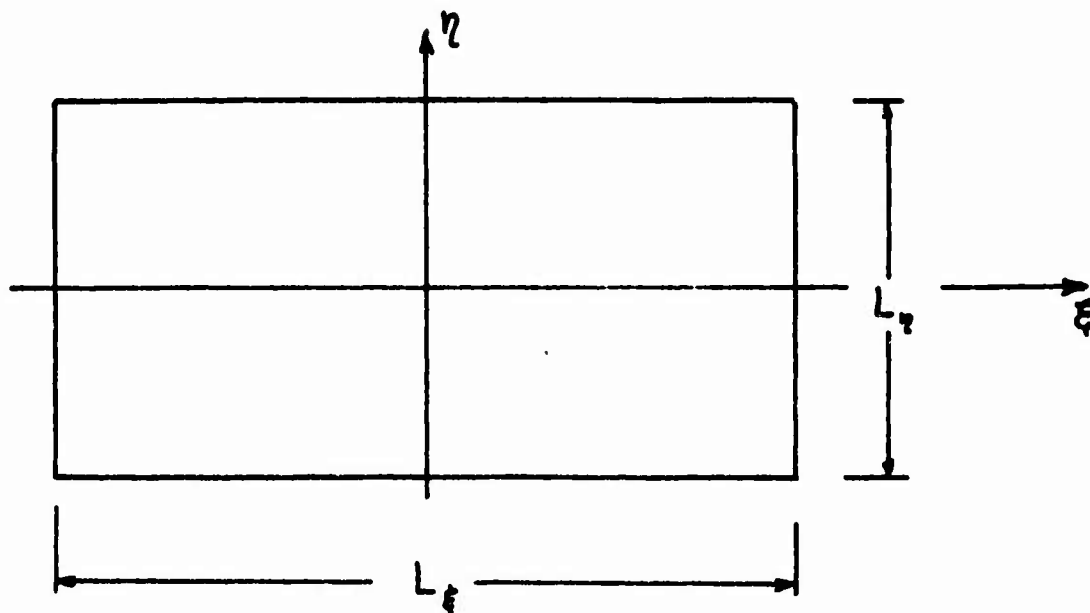


Figure 3
Transformed Plate

In the book, charts are drawn for the values,

$$e = 2.00; 1.25; 1.00; 0.80; 0.50.$$

$$x = 0.80; 0.40; 0.$$

The dimension L , from Eqs. (63) and (64) is chosen by the user of the influence surfaces, such that the length of the shorter side of the transformed plate becomes either 20, or 16, or 10, corresponding to the given e . Some results of this procedure are shown in the following chapter.

PART 3 - FLEXURAL BEHAVIOR

In this part, the bending problem, for various combinations of edge supports and loadings, is considered. The chapter is subdivided into sections according to the support conditions, with further subdivisions made for different conditions of loading. In the accompanying drawings, the following notation is used for edge supports:

- (C) = clamped,
- (F) = free,
- (S) = simply supported.

3.1 Simple Supports

The basic differential equation (31) of the orthotropic plate is recalled:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = P,$$

where the loading term is a function of x and y ,

$$P = f(x, y), \quad (67)$$

and it can represent any type of transverse loading on the plate. The double Fourier series (Navier) form of this function is:

$$f(x,y) = \sum_m \sum_n A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \quad (68)$$

where

m = number of sinusoidal half-waves in the x-direction,

n = number of sinusoidal half-waves in the y-direction.

The total deflection of the plate shown in Figure 4 is then calculated as the sum of the partial deflections produced by the partial sinusoidal loadings of Eq. (68).

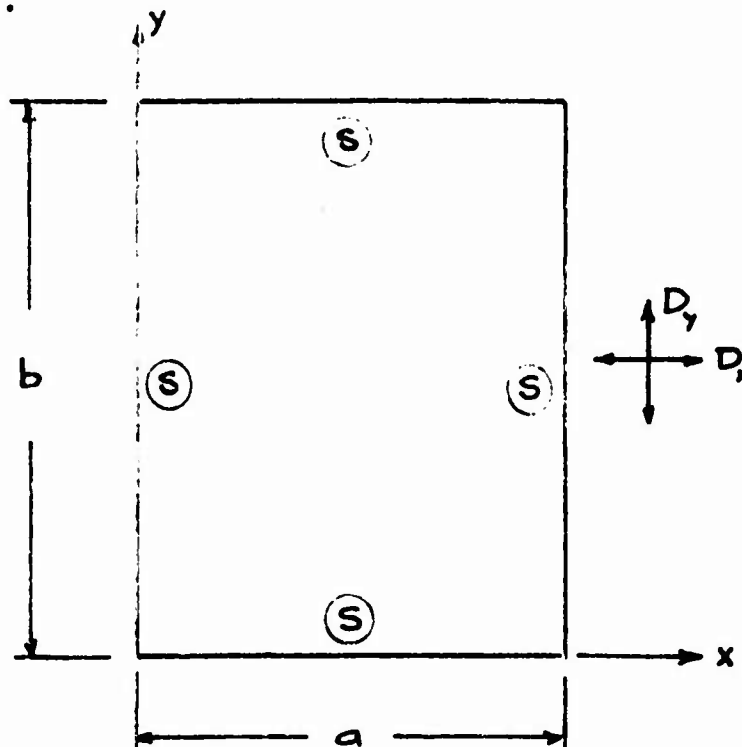


Figure 4
Simply Supported Plate

The general expression for the deformation of a simply supported rectangular orthotropic plate, in the Navier form, is given by:

$$w = \frac{1}{\pi^4} \sum_m \sum_n A_{mn} \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{D_x \left(\frac{m^4}{a^4} \right) + 2D_{xy} \left(\frac{n^2}{a^2 b^2} \right) + D_y \left(\frac{n^4}{b^4} \right)} \quad (69)$$

An alternate form of solution (discussed in Part 2) is the single-series expression attributed to Lévy. This solution is based upon the equation,

$$w = \sum_m Y_m \sin \frac{m\pi x}{a} \quad , \quad (70)$$

in which Y_m is a function of y alone and the b - edges ($x = 0, a$) are simply supported.

Both methods, in addition to any of the additional techniques described in Part 2, will be utilized here whenever they are applicable.

3.1.1 Uniformly Distributed Load over Entire Surface

The case of a simply supported plate, acted upon by a load, p_0 , uniformly distributed over its entire surface is shown in Figure 5 and considered here.

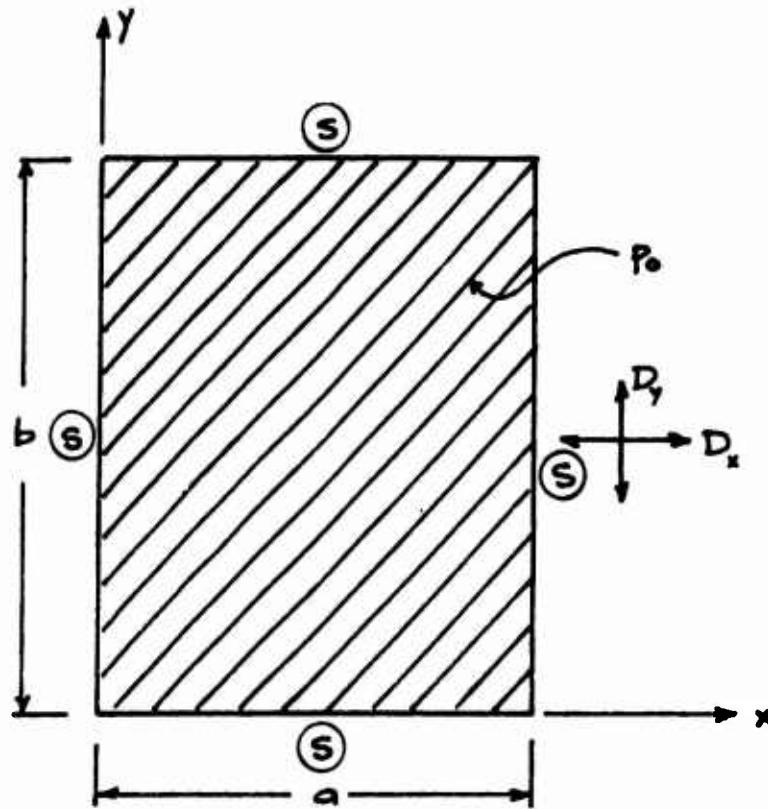


Figure 5
Simple Supports - Uniform Load

The Navier solution, as given in Reference 97, is first utilized. To evaluate the coefficient A_{mn} of Eq. (69), it is found from Eq. (68) that

$$A_{mn} = \frac{4}{ab} \int_0^a \int_0^b f(x,y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dy dx. \quad (71)$$

For the uniform loading,

$$f(x,y) = p_0, \quad (72)$$

and the coefficient thus becomes:

$$\left. \begin{aligned} A_{mn} &= \frac{4p_0}{ab} \int_0^a \int_0^b \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dy dx \\ &= \frac{16p_0}{\pi^2 mn} \end{aligned} \right\} (73)$$

The substitution of Eq. (73) into Eq. (69) yields the deformation expression for a uniformly loaded, simply supported plate:

$$w = \frac{16p_0}{\pi^6} \sum_m \sum_n \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn \left(D_x \frac{m^4}{a^4} + 2D_y \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right)} \quad (74)$$

Using the Lévy form of solution, Ando (Ref. 209) writes the deformation equation:

$$w = \frac{4p_0 a^4}{D_x \pi^5} \sum_m \frac{(1 + \tilde{A}_m C + \tilde{B}_m S)}{m^5} \sin \frac{m\pi x}{a}, \quad (75)$$

where

$$\tilde{A}_m = - \frac{C_0 + \frac{\xi}{\sqrt{1-\xi^2}} S_0}{\sinh^2 \alpha_m t + \cos^2 \alpha_m u},$$

$$\hat{B}_m = \frac{\xi C_0 - S_0}{\sinh^2 \alpha_m t + \cos^2 \alpha_m u} ,$$

$$C = \cosh \frac{m\pi t}{a} y \cos \frac{m\pi u}{a} y ,$$

$$S = \sinh \frac{m\pi t}{a} y \sin \frac{m\pi u}{a} y$$

$$t = \sqrt{\frac{\sqrt{D_x D_y} + D_{xy}}{2}} ,$$

$$u = \sqrt{\frac{\sqrt{D_x D_y} - D_{xy}}{2}} ,$$

$$\xi = \frac{D_{xy}}{\sqrt{D_x D_y}} ,$$

$$\alpha_m = \frac{m\pi y}{a} ,$$

0 = subscript indicating initial condition.

The same problem is solved by Hess and Vinson (Ref. 34) using a perturbation technique. The basic equation is written in the form:

$$s^4 - 2 \frac{D_{xy}}{D_x} s^2 + \frac{D_y}{D_x} = 0 , \quad (76)$$

which has the roots,

$$s = \pm \sqrt{\frac{D_{xy}}{D_x} \pm \sqrt{\left(\frac{D_{xy}}{D_x}\right)^2 - \frac{D_y}{D_x}}} . \quad (77)$$

There are, thus, three cases of solution, which are dependent upon the elastic properties of the orthotropic plate:

CASE I:

$$\left(\frac{D_{xy}}{D_x}\right)^2 > \frac{D_y}{D_x} .$$

The deformation function for this case is given by:

$$w = \sum_{n=1,3,\dots} \left\{ \frac{s_1^2}{s_1^2 - s_2^2} \left[\cosh s_1 \lambda_n x + \left(\frac{1 - \cosh s_1 \lambda_n a}{\sinh s_1 \lambda_n a} \right) \cdot \sinh s_1 \lambda_n x \right] + \frac{s_2^2}{s_2^2 - s_1^2} \left[\cosh s_2 \lambda_n x + \left(\frac{1 - \cosh s_2 \lambda_n a}{\sinh s_2 \lambda_n a} \right) \cdot \sinh s_2 \lambda_n x \right] + 1 \right\} \Phi \sin \lambda_n y . \quad (78)$$

CASE II:

$$\left(\frac{D_{xy}}{D_x}\right)^2 = \frac{D_y}{D_x} .$$

$$\begin{aligned}
 W = \sum_{n=1,3,\dots} \left\{ \left[-1 + \frac{\lambda_n s_3 x}{2} \left(\frac{1 - \cosh \lambda_n s_3 a}{\sinh \lambda_n s_3 a} \right) \right] \cosh \lambda_n s_3 x \right. \\
 \left. - \left[\frac{\cosh \lambda_n s_3 a - 1}{\sinh \lambda_n s_3 a} \left(\frac{a \lambda_n s_3}{2} - \sinh \lambda_n s_3 a \right) \right. \right. \\
 \left. \left. - \frac{\lambda_n s_3 x}{2} \right] \sinh \lambda_n s_3 x \right\} \Phi \sin \lambda_n x.
 \end{aligned} \tag{79}$$

CASE III:

$$\left(\frac{D_{xy}}{D_x} \right)^2 < \frac{D_y}{D_x}.$$

$$\begin{aligned}
 W = \sum_{n=1,3,\dots} \frac{4 p_0 b^4}{n^5 \pi^5 D_y} \left\{ \left[-\cos \lambda_n s_5 x + \frac{(\cos \lambda_n s_5 a - \cosh \lambda_n s_4 a)}{(\cosh^2 \lambda_n s_4 a - \cos^2 \lambda_n s_5 a)} \right. \right. \\
 \cdot \left(\sin \lambda_n s_5 a + \frac{s_4^2 - s_5^2}{2 s_4 s_5} \sinh \lambda_n s_4 a \right) \sin \lambda_n s_5 x \Big] \cosh \lambda_n s_4 x \\
 + \left[\frac{(\cos \lambda_n s_5 a - \cosh \lambda_n s_4 a)}{(\cosh^2 \lambda_n s_4 a - \cos^2 \lambda_n s_5 a)} (-\sinh \lambda_n s_4 a + \frac{s_4^2 - s_5^2}{2 s_4 s_5} \sin \lambda_n s_5 a) \right. \\
 \cdot \left. \cos \lambda_n s_5 x + \frac{s_4^2 - s_5^2}{2 s_4 s_5} \sin \lambda_n s_5 x \right] \sinh \lambda_n s_4 x \Big\} \sin \lambda_n y.
 \end{aligned} \tag{80}$$

In the above equations, the roots:

$$s_1 = \sqrt{\frac{D_{xy}}{D_x}} + \sqrt{\left(\frac{D_{xy}}{D_x}\right)^2 - \frac{D_y}{D_x}} ,$$

$$s_2 = \sqrt{\frac{D_{xy}}{D_x}} - \sqrt{\left(\frac{D_{xy}}{D_x}\right)^2 - \frac{D_y}{D_x}} ,$$

$$s_3 = \sqrt{\frac{D_{xy}}{D_x}} ,$$

$$s_4 = \sqrt{\frac{1}{2} \left[\sqrt{\frac{D_y}{D_x}} + \frac{D_{xy}}{D_x} \right]} ,$$

$$s_5 = \sqrt{\frac{1}{2} \left[\sqrt{\frac{D_y}{D_x}} - \frac{D_{xy}}{D_x} \right]} .$$

The coordinates and elastic constants are taken according to Figure 5, and the parameter,

$$\lambda = \frac{\pi n}{b} .$$

Using these equations as models, design curves are plotted using the dimensionless parameters,

$$\frac{w_{max} D_y}{P_0 b^4} , \quad \frac{M_{x(max)}}{P_0 b^2} , \quad \frac{M_{y(max)}}{P_0 b^2} ,$$

as functions of the three independent variables,

$$\frac{D_y}{D_x} , \quad \frac{D_{xy}}{D_x} , \quad \frac{b}{a} .$$

3.1.2 Uniformly Distributed Load over Part of Surface

A simply supported plate, acted upon by a uniformly distributed load over a portion of its surface (Fig. 6) is now considered.

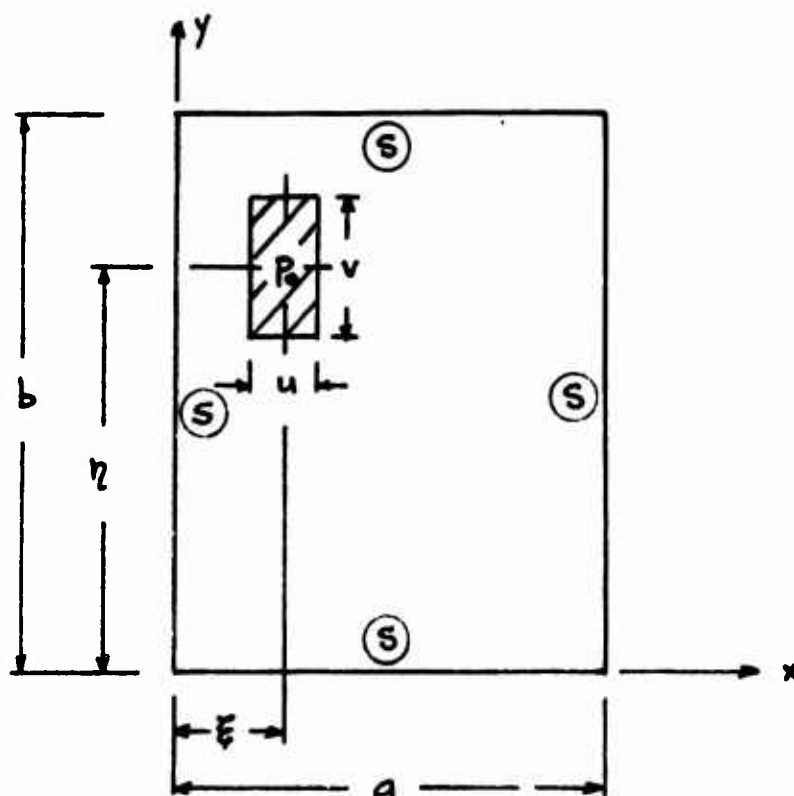


Figure 6
Simple Supports - Partial Load (I)

First, the Navier solution is applied. Recalling the equation giving the loading coefficient (Eq. (68)) and applying the proper conditions:

$$\begin{aligned}
 A_{mn} &= \frac{4P_0}{ab} \int_{\xi-\eta/2}^{\xi+\eta/2} \int_{\eta-\eta/2}^{\eta+\eta/2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dy dx \\
 &= \frac{16P_0}{\pi^2 mn} \sin \frac{m\pi \xi}{a} \sin \frac{n\pi \eta}{b} \sin \frac{m\pi u}{2a} \sin \frac{n\pi v}{2b}
 \end{aligned} \quad (81)$$

Substitution of this into Eq. (69) gives the deformation expression:

$$w = \frac{16P_0}{\pi^6} \sum_m \sum_n \frac{\sin \frac{m\pi \xi}{a} \sin \frac{n\pi \eta}{b} \sin \frac{m\pi u}{2a} \sin \frac{n\pi v}{2b} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn \left(D_x \frac{m^4}{a^4} + 2D_{xy} \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4} \right)} \quad (82)$$

Next, the Lévy-type solution, as given by Ando (Ref. 209) is examined. The x-coordinate is changed (Fig. 7) to coincide with the center of the plate.

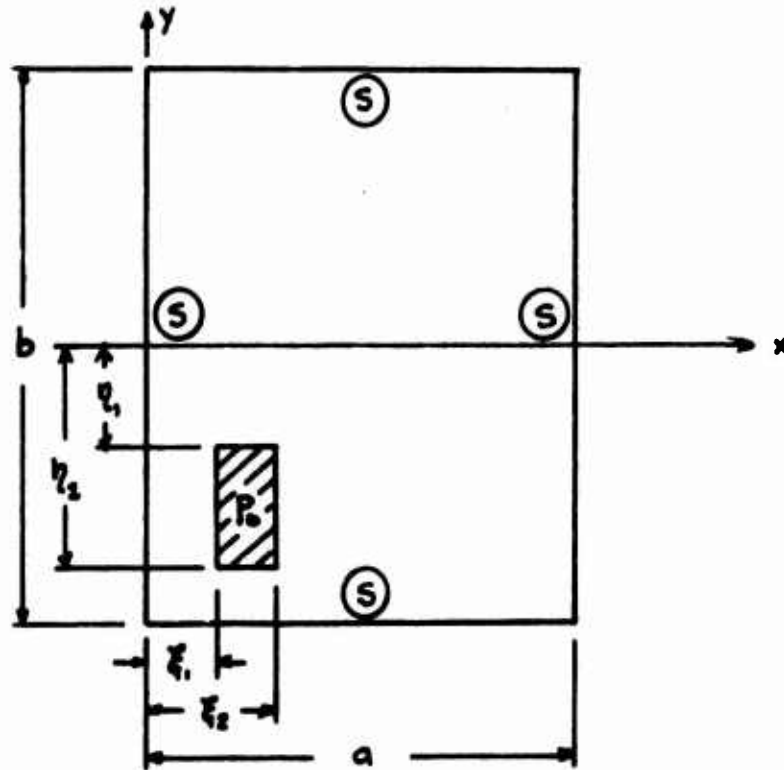


Figure 7
Simple Supports - Partial Load (II)

Deflection expressions are written for three regions of the plate:

CASE I: $\eta_1 \leq y \leq \eta_2$:

$$w_I = \sum_m \left(A_m + C_1 \cosh \frac{m\pi\lambda y}{a} + C_2 \sinh \frac{m\pi\lambda y}{a} + C_3 \cosh \frac{m\pi\bar{\lambda} y}{a} + C_4 \sinh \frac{m\pi\bar{\lambda} y}{a} \right) \sin \frac{m\pi x}{a}. \quad (83)$$

CASE II: $\eta_2 \leq y \leq b/2$:

$$w_{II} = \sum_m \left(C_5 \cosh \frac{m\pi\lambda y}{a} + C_6 \sinh \frac{m\pi\lambda y}{a} + C_7 \cosh \frac{m\pi\bar{\lambda} y}{a} + C_8 \sinh \frac{m\pi\bar{\lambda} y}{a} \right) \sin \frac{m\pi x}{a}. \quad (84)$$

CASE III: $-b/2 \leq y \leq \eta_1$:

$$w_{III} = \sum_m \left(C_9 \cosh \frac{m\pi\lambda y}{a} + C_{10} \sinh \frac{m\pi\lambda y}{a} + C_{11} \cosh \frac{m\pi\bar{\lambda} y}{a} + C_{12} \sinh \frac{m\pi\bar{\lambda} y}{a} \right) \sin \frac{m\pi x}{a}. \quad (85)$$

The constants of the above equations are evaluated from boundary values:

$$C_1 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\cosh \frac{m\pi\lambda\eta_1}{a} + \cosh \frac{m\pi\lambda\eta_2}{a} \right. \right. \\ \left. \left. - \left(\sinh \frac{m\pi\lambda\eta_1}{a} - \sinh \frac{m\pi\lambda\eta_2}{a} \right) \tanh \frac{m\pi\lambda y}{a} \right] \right\},$$

$$C_2 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\sinh \frac{m\pi\lambda\eta_1}{a} + \sinh \frac{m\pi\lambda\eta_2}{a} \right. \right. \\ \left. \left. - \left(\cosh \frac{m\pi\lambda\eta_1}{a} - \cosh \frac{m\pi\lambda\eta_2}{a} \right) \coth \frac{m\pi\lambda y}{a} \right] \right\},$$

$$C_3 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\cosh \frac{m\pi\bar{\lambda}\eta_1}{a} + \cosh \frac{m\pi\bar{\lambda}\eta_2}{a} \right. \right. \\ \left. \left. - \left(\sinh \frac{m\pi\bar{\lambda}\eta_1}{a} - \sinh \frac{m\pi\bar{\lambda}\eta_2}{a} \right) \tanh \frac{m\pi\bar{\lambda}y}{a} \right] \right\},$$

$$C_4 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\sinh \frac{m\pi\bar{\lambda}\eta_1}{a} + \sinh \frac{m\pi\bar{\lambda}\eta_2}{a} \right. \right. \\ \left. \left. - \left(\cosh \frac{m\pi\bar{\lambda}\eta_1}{a} - \cosh \frac{m\pi\bar{\lambda}\eta_2}{a} \right) \coth \frac{m\pi\bar{\lambda}y}{a} \right] \right\},$$

$$C_5 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\cosh \frac{m\pi\lambda\eta_1}{a} - \cosh \frac{m\pi\lambda\eta_2}{a} \right. \right. \\ \left. \left. - \left(\sinh \frac{m\pi\lambda\eta_1}{a} - \sinh \frac{m\pi\lambda\eta_2}{a} \right) \tanh \frac{m\pi\lambda y}{a} \right] \right\},$$

$$C_6 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\sinh \frac{m\pi\lambda\eta_1}{a} - \sinh \frac{m\pi\lambda\eta_2}{a} \right. \right. \\ \left. \left. - \left(\cosh \frac{m\pi\lambda\eta_1}{a} - \cosh \frac{m\pi\lambda\eta_2}{a} \right) \coth \frac{m\pi\lambda y}{a} \right] \right\},$$

$$C_7 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\cosh \frac{m\pi\bar{\lambda}\eta_1}{a} - \cosh \frac{m\pi\bar{\lambda}\eta_2}{a} \right. \right. \\ \left. \left. - \left(\sinh \frac{m\pi\bar{\lambda}\eta_1}{a} - \sinh \frac{m\pi\bar{\lambda}\eta_2}{a} \right) \tanh \frac{m\pi\bar{\lambda}y}{a} \right] \right\},$$

$$- \left(\sinh \frac{m\pi \bar{\lambda} \eta_1}{a} - \sinh \frac{m\pi \bar{\lambda} \eta_2}{a} \right) \tanh \frac{m\pi \bar{\lambda} y}{a} \Bigg\},$$

$$C_8 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[\sinh \frac{m\pi \bar{\lambda} \eta_1}{a} - \sinh \frac{m\pi \bar{\lambda} \eta_2}{a} \right. \right. \\ \left. \left. - \left(\cosh \frac{m\pi \bar{\lambda} \eta_1}{a} - \cosh \frac{m\pi \bar{\lambda} \eta_2}{a} \right) \coth \frac{m\pi \bar{\lambda} y}{a} \right] \right\},$$

$$C_9 = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[-\cosh \frac{m\pi \lambda \eta_1}{a} + \cosh \frac{m\pi \lambda \eta_2}{a} \right. \right. \\ \left. \left. - \left(\sinh \frac{m\pi \lambda \eta_1}{a} - \sinh \frac{m\pi \lambda \eta_2}{a} \right) \tanh \frac{m\pi \lambda y}{a} \right] \right\},$$

$$C_{10} = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[-\sinh \frac{m\pi \lambda \eta_1}{a} + \sinh \frac{m\pi \lambda \eta_2}{a} \right. \right. \\ \left. \left. - \left(\cosh \frac{m\pi \lambda \eta_1}{a} - \cosh \frac{m\pi \lambda \eta_2}{a} \right) \coth \frac{m\pi \lambda y}{a} \right] \right\},$$

$$C_{11} = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[-\cosh \frac{m\pi \bar{\lambda} \eta_1}{a} + \cosh \frac{m\pi \bar{\lambda} \eta_2}{a} \right. \right. \\ \left. \left. - \left(\sinh \frac{m\pi \bar{\lambda} \eta_1}{a} - \sinh \frac{m\pi \bar{\lambda} \eta_2}{a} \right) \tanh \frac{m\pi \bar{\lambda} y}{a} \right] \right\},$$

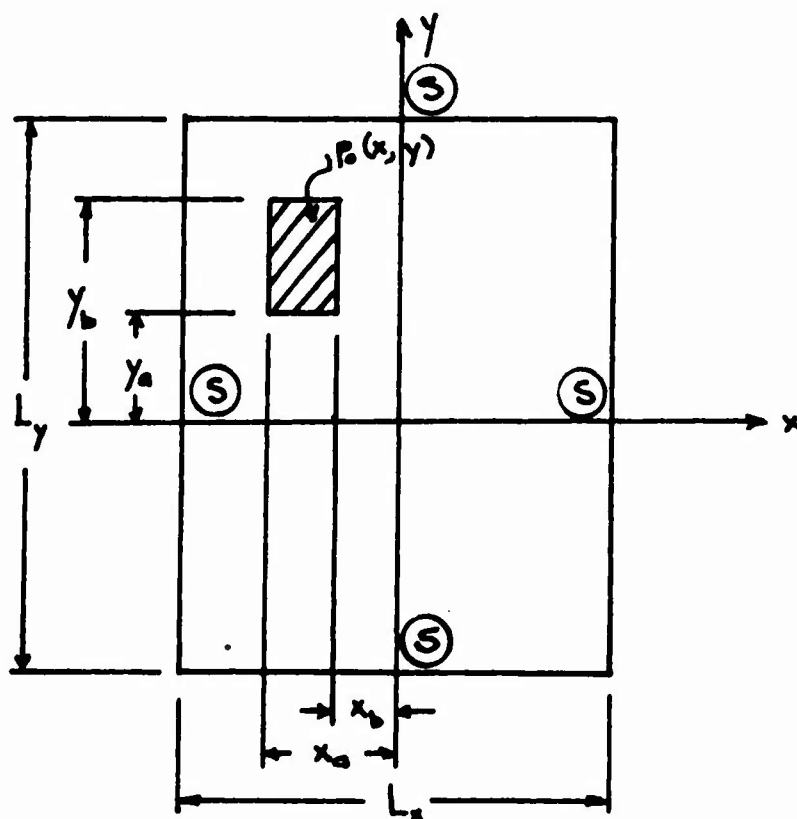
$$C_{12} = \frac{A_m}{2} \left\{ \frac{\bar{\lambda}^2}{\lambda^2 - \bar{\lambda}^2} \left[-\sinh \frac{m\pi \bar{\lambda} \eta_1}{a} + \sinh \frac{m\pi \bar{\lambda} \eta_2}{a} \right. \right. \\ \left. \left. - \left(\cosh \frac{m\pi \bar{\lambda} \eta_1}{a} - \cosh \frac{m\pi \bar{\lambda} \eta_2}{a} \right) \coth \frac{m\pi \bar{\lambda} y}{a} \right] \right\},$$

$$\lambda = \sqrt{\frac{\sqrt{D_x D_y} + D_{xy}}{2}} + i \sqrt{\frac{\sqrt{D_x D_y} - D_{xy}}{2}},$$

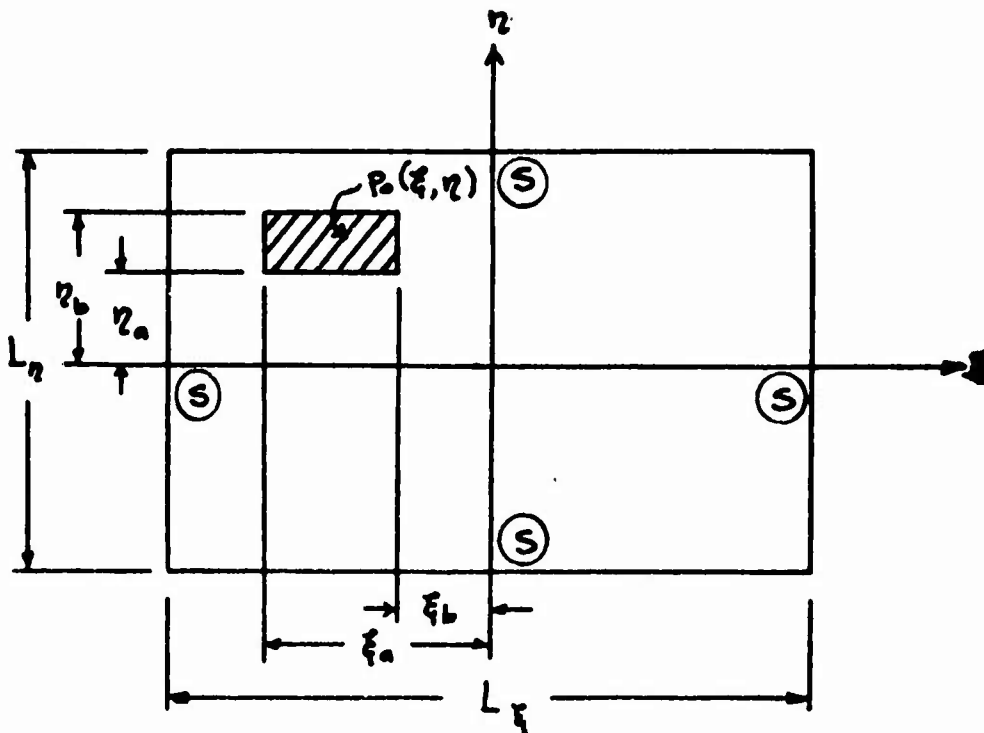
$$\bar{\lambda} = \sqrt{\frac{\sqrt{D_x D_y} + D_{xy}}{2}} - i \sqrt{\frac{\sqrt{D_x D_y} - D_{xy}}{2}},$$

$$\begin{aligned}
 A_m &= -\frac{2a^3 p_0}{D_x m^4 \pi^4} \int_{\xi_1}^{\xi_2} \sin \frac{m\pi \xi}{a} d\xi \\
 &= \frac{2a^4 p_0}{D_x m^5 \pi^5} \left(\cos \frac{m\pi \xi_1}{a} - \cos \frac{m\pi \xi_2}{a} \right).
 \end{aligned}$$

In deriving the influence surfaces of their book of tables (Ref. 57), Krug and Stein make use of a transformation from the real structure (Fig. 8a) to a "new" structure (Fig. 8b).



a) Real Structure



b) Transformed Structure

Figure 8
Simple Supports - Partial Load (III)

The method of transformation is given in Eqs. (62) through (65), and the bending moments in the plate are:

$$\left. \begin{aligned} M_x &= \frac{P_0 L^2}{8\pi} \int_{\xi_a}^{\xi_b} \int_{\eta_a}^{\eta_b} M_{\xi}(\xi, \eta) d\xi d\eta, \\ M_y &= \frac{P_0 L^2}{8\pi\sqrt{\lambda}} \int_{\xi_a}^{\xi_b} \int_{\eta_a}^{\eta_b} M_{\eta}(\xi, \eta) d\xi d\eta. \end{aligned} \right\} (86)$$

Numerical values given by these integrals, for given geometry and constants of orthotropy, are given in the tables.

3.1.3 Concentrated Force

The next problem to be considered here is that of the simply supported plate under the action of a concentrated load, P .

The writer applied the Navier form of solution to this case in Reference 309. This is considered first (Figure 9).

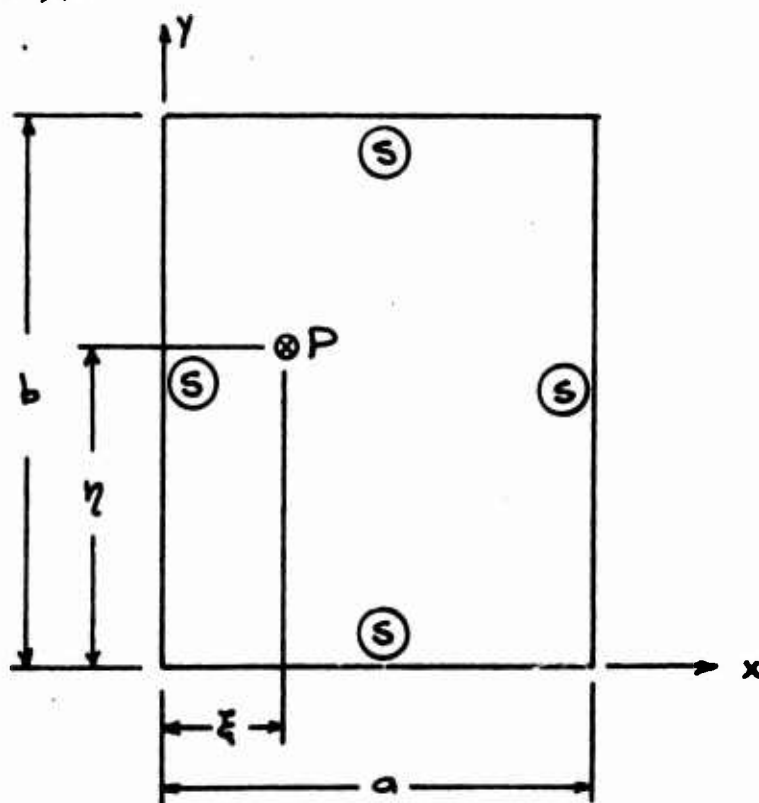


Figure 9
Simple Supports - Concentrated Load (I)

The load coefficient for the double-series solution, from Eq. (68), is written:

$$A_{mn} = \frac{4P}{ab} \sin \frac{m\pi\xi}{a} \sin \frac{n\pi\eta}{b} \quad (87)$$

The deformation expression is derived from the substitution of this quantity into the general equation (69):

$$w = \frac{4P}{ab\pi^4} \sum_m \sum_n \frac{\sin \frac{m\pi\xi}{a} \sin \frac{n\pi\eta}{b} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{D_x \frac{m^4}{a^4} + 2D_{xy} \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4}} \quad (88)$$

Ando (Ref. 209) solves the problem using the single series solution. Using a coordinate system chosen such that the x-axis is located at the center, rather than the edge, of the plate (Fig. 10), he writes two expressions for the deformation:

CASE I: $\eta \leq y \leq b/2$:

$$w_I = \sum_m \left[C_1 \cosh \frac{m\pi\lambda y}{a} + C_2 \sinh \frac{m\pi\lambda y}{b} + C_1 \cosh \frac{m\pi\bar{\lambda} y}{a} + C_2 \sinh \frac{m\pi\bar{\lambda} y}{b} \right] \sin \frac{m\pi x}{a}, \quad (89)$$

CASE II:

$$w_z = \sum_m \left[C_3 \cosh \frac{m\pi\lambda y}{a} + C_4 \sinh \frac{m\pi\lambda y}{b} + C_3 \cosh \frac{m\pi\bar{\lambda} y}{a} + C_4 \sinh \frac{m\pi\bar{\lambda} y}{b} \right] \sin \frac{m\pi x}{a}. \quad (90)$$

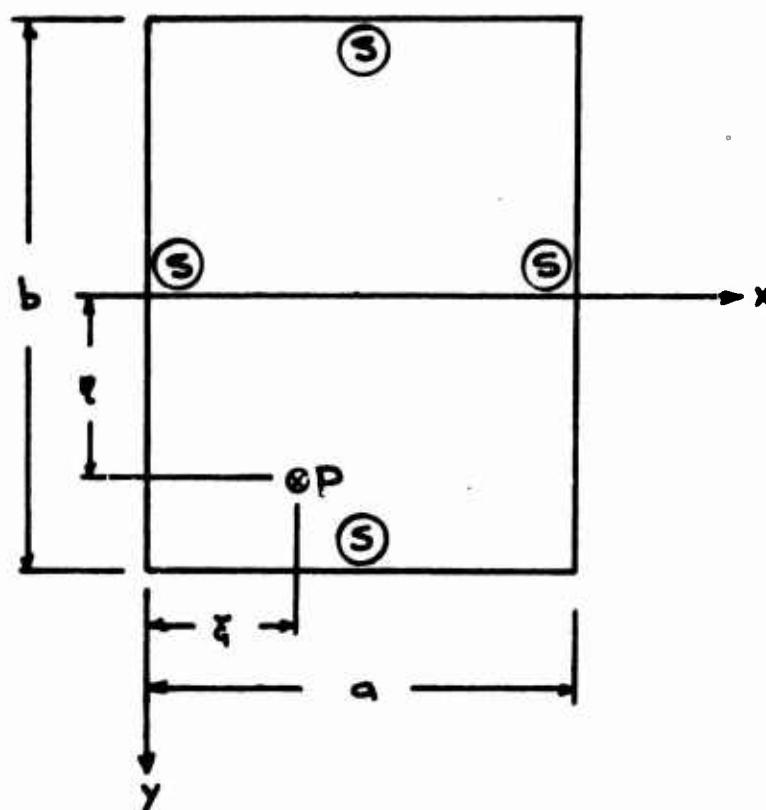


Figure 10
Simple Supports - Concentrated Load (II)

The constants of Eqs. (89) and (90) are the following:

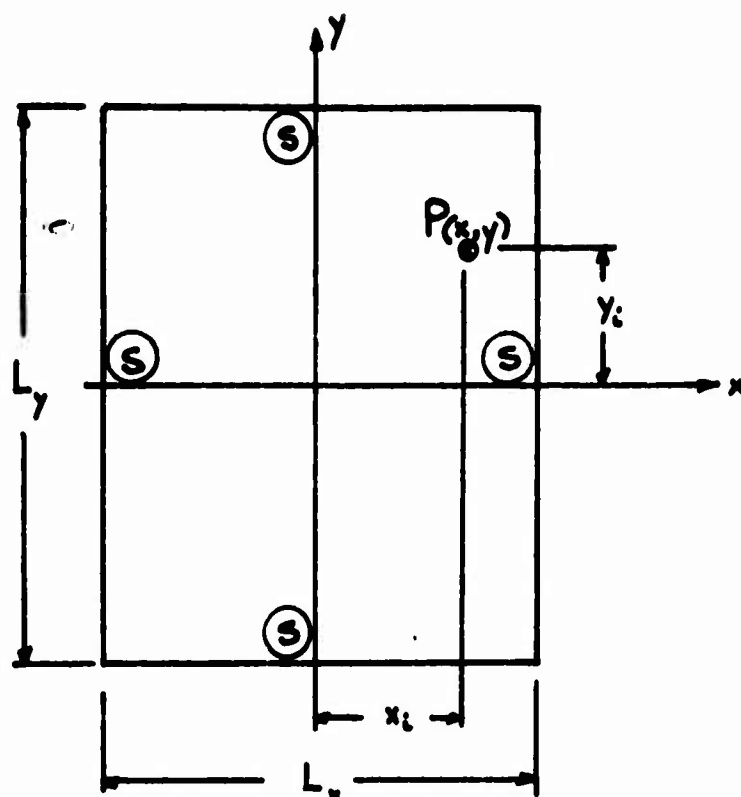
$$C_1 = \frac{Pa^2 \lambda \bar{\lambda}^2}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[-\sinh \frac{m\pi \lambda \eta}{a} - \left(\cosh \frac{m\pi \lambda \eta}{a} \cdot \tanh \frac{m\pi \lambda \gamma}{a} \right) \right] \sin \frac{m\pi \xi}{a},$$

$$C_2 = \frac{Pa^2 \lambda \bar{\lambda}^2}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[\cosh \frac{m\pi \lambda \eta}{a} + \left(\sinh \frac{m\pi \lambda \eta}{a} \cdot \coth \frac{m\pi \lambda \gamma}{a} \right) \right] \sin \frac{m\pi \xi}{a},$$

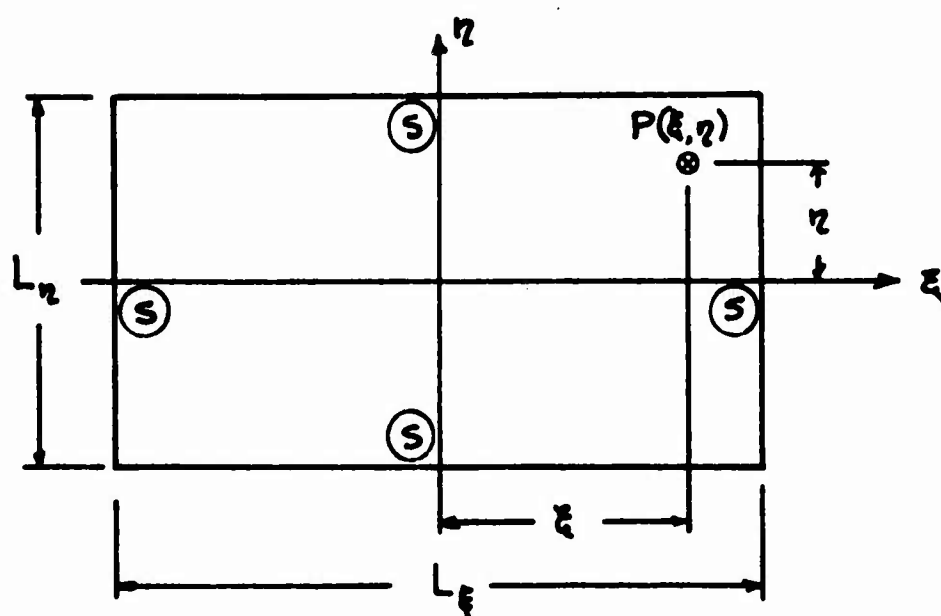
$$C_3 = \frac{Pa^2 \lambda \bar{\lambda}^2}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[\sinh \frac{m\pi \lambda \eta}{a} - \left(\cosh \frac{m\pi \lambda \eta}{a} \cdot \tanh \frac{m\pi \lambda \gamma}{a} \right) \right] \sin \frac{m\pi \xi}{a},$$

$$C_4 = \frac{Pa^2 \lambda \bar{\lambda}^2}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[-\cosh \frac{m\pi \lambda \eta}{a} + \left(\sinh \frac{m\pi \lambda \eta}{a} \cdot \coth \frac{m\pi \lambda \gamma}{a} \right) \right] \sin \frac{m\pi \xi}{a}.$$

As in the previous case, Krug and Stein (Ref. 57) make use of the transformation of coordinates, as illustrated in Figures 11a and 11b.



a) Real Structure



b) Transformed Structure

Figure 11
Simple Supports - Concentrated Load (III)

Using the transformations given by Eqs. (62) through (65), the moments are given:

$$\left. \begin{aligned} M_x &= \frac{1}{8\pi} \sqrt{\frac{D_x}{D_y}} P(\xi, \eta) M_{\xi}(\xi, \eta), \\ M_y &= \frac{1}{8\pi} \sqrt{\frac{D_x}{D_y}} P(\xi, \eta) M_{\eta}(\xi, \eta). \end{aligned} \right\} (91)$$

Obviously, superposition is used for the calculation of deflections in the first two solutions, when more than one concentrated load is acting, and the latter solution (Eq. (91)) can be rewritten in the form:

$$M_x = \frac{1}{8\pi} \sqrt{\frac{D_x}{D_y}} \sum_i P(\xi_i, \eta_i) M_{\xi}(\xi_i, \eta_i). \quad (92)$$

3.1.4 Line Loading

In solving the problem of the simply supported plate under the action of a linear load parallel to the x-axis (Figure 12), Ando (Ref. 209) uses Eqs. (89) and (90) to give the deformation for the ranges $\eta \leq y \leq b/2$ and $y \leq \eta$, respectively. The constants are evaluated as follows:

$$C_i = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\sinh \frac{m\pi\lambda\eta}{a} - \left(\cosh \frac{m\pi\lambda\eta}{a} \tanh \frac{m\pi\lambda y}{a} \right) \right],$$

$$C_2 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[\cosh \frac{m\pi\lambda\eta}{a} + \left(\sinh \frac{m\pi\lambda\eta}{a} \coth \frac{m\pi\lambda\gamma}{a} \right) \right],$$

$$C_3 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[\sinh \frac{m\pi\lambda\eta}{a} - \left(\cosh \frac{m\pi\lambda\eta}{a} \tanh \frac{m\pi\lambda\gamma}{a} \right) \right],$$

$$C_4 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\cosh \frac{m\pi\lambda\eta}{a} + \left(\sinh \frac{m\pi\lambda\eta}{a} \coth \frac{m\pi\lambda\gamma}{a} \right) \right],$$

and

$$f_m = \frac{2a^2}{D_x m^3 \pi^3} \int_{\xi_1}^{\xi_2} F(\xi) \sin \frac{m\pi\xi}{a} d\xi.$$

The load, $F(\xi)$ is represented as a single trigonometric series.

Should one consider the special case where the load acts on the x-axis ($\eta = 0$), the constants simplify to:

$$C_1 = -\frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \tanh \frac{m\pi\lambda\gamma}{a} = C_3,$$

$$C_2 = \frac{f_m \lambda \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} = -C_4.$$

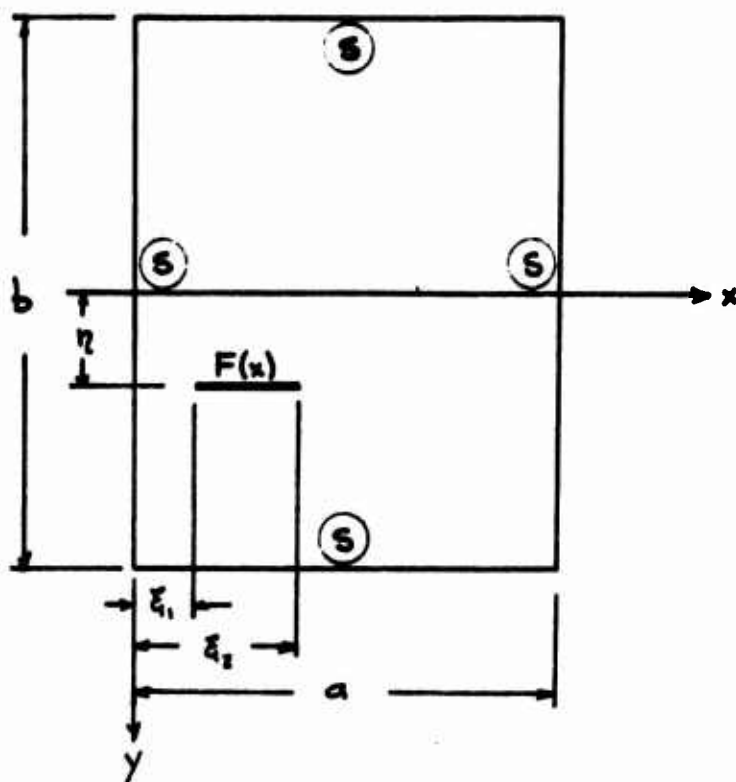


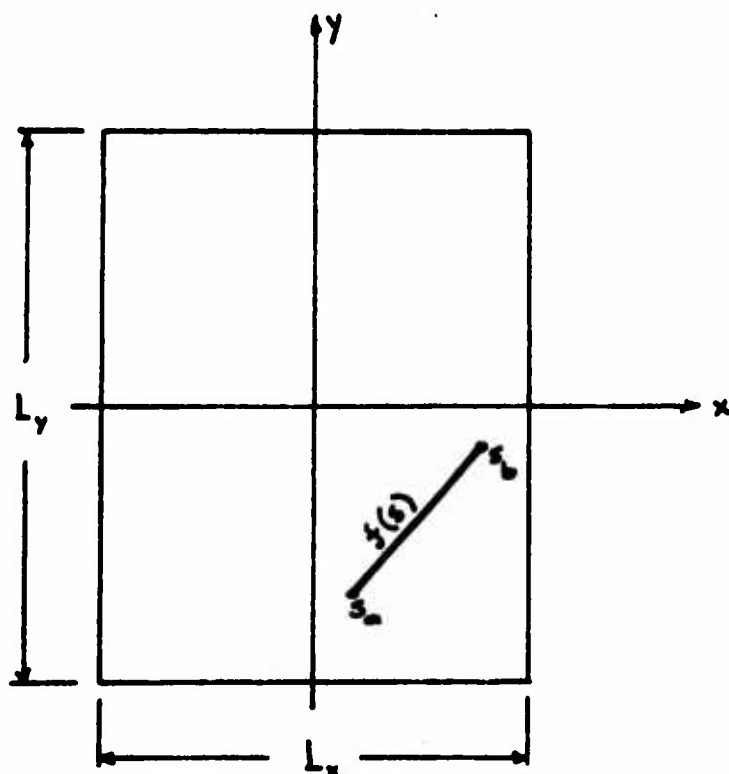
Figure 12
Simple Supports - Line Load (I)

A more general inclination of the line load is considered by Krug and Stein (Ref. 57). As previously described for other loadings, the coordinates are transformed from those of Figure 13a to those of Figure 13b, according to Eqs. (62) through (65).

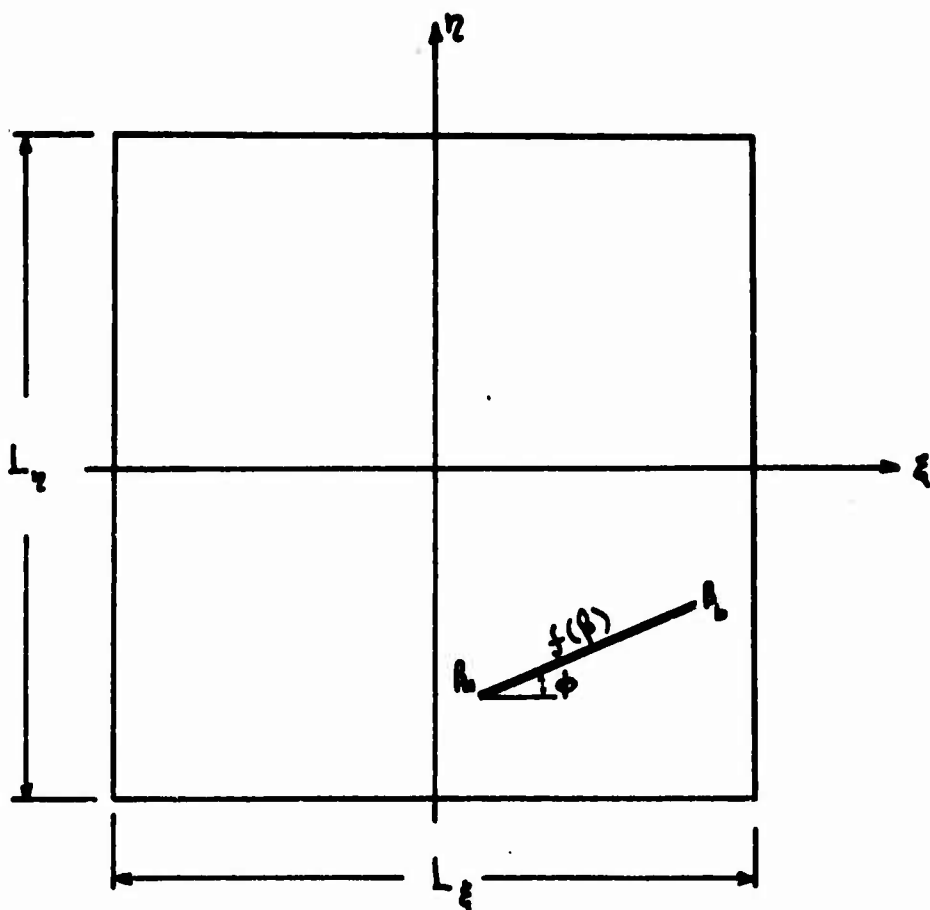
The bending moment expressions are then given for the general case of variable loading, f :

$$M_x = \frac{1}{8\pi} \sqrt{\frac{D_x}{D_y}} \sqrt{1 + \frac{\tan^2 \phi}{\sqrt{(D_x/D_y)}}} (\cos \phi) L \int_{\beta_a}^{\beta_b} f(\beta) M_f(\beta) d\beta, \quad (93)$$

$$M_y = \frac{1}{8\pi} \sqrt{\frac{D_x}{I}} \sqrt{1 + \frac{\tan^2 \phi}{\sqrt{(D_x/D_y)}}} (\cos \phi) L \int_{\beta_a}^{\beta_b} f(\beta) M_f(\beta) d\beta. \quad (94)$$



a) Real Structure



b) Transformed Structure

Figure 13
Simple Supports - Line Load (II)

For the special case of a load uniformly distributed along a line, of course, $f(\beta) = f$, and it is placed in front of the integral sign.

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 十五
 十六
 十七
 十八
 十九
 二十

Using the transformations,

$$\bar{y} = y \sqrt{\frac{D_x}{D_y}} \quad , \quad \bar{b} = b \sqrt{\frac{D_x}{D_y}} \quad , \quad (95)$$

the boundary conditions are:

$$\left. \begin{aligned} w(x, 0) = w(x, \bar{b}) = w(0, \bar{y}) = w(a, \bar{y}) = 0, \\ \frac{\partial^2 w}{\partial x^2} \Big|_{(0, \bar{y})} = \frac{\partial^2 w}{\partial x^2} \Big|_{(a, \bar{y})} = \frac{\partial^2 w}{\partial y^2} \Big|_{(x, 0)} = 0, \\ M(x, \bar{b}) = M. \end{aligned} \right\} (96)$$

If the perturbation solution is limited to cases where the rigidity ratios (D_y/D_x) and (D_{xy}/D_x) are nearly equal, the basic equation (31) of the orthotropic plate is rewritten:

$$\frac{\partial^4 w}{\partial x^4} - \alpha \frac{\partial^4 w}{\partial x^2 \partial \bar{y}^2} + \frac{\partial^4 w}{\partial \bar{y}^4} = \frac{p}{D_x} \quad , \quad (97)$$

where

$$\alpha = 2 \left(1 - \frac{D_{xy}}{\sqrt{D_x D_y}} \right) = 2 \left(1 - \frac{(D_{xy}/D_x)}{\sqrt{(D_y/D_x)}} \right) .$$

The solution is taken as a series in powers of :

$$w = \sum_{n=0}^{\infty} \Phi_n(x, \bar{y}) \alpha^n \quad , \quad (98)$$

which makes Eq. (97) become:

$$\nabla^4 \Phi_n + \sum_{m=1}^{\infty} \left[\nabla^4 \Phi_m - \frac{\partial^4 \Phi_{m-1}}{\partial x^2 \partial \bar{y}^2} \right] \alpha^n = \frac{p}{D_x} \quad , \quad (99)$$

where the operator,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \bar{y}^2}.$$

Since the curvature vanishes along edge $y = \bar{b}$, the moment,

$$M(x, \bar{b}) = M = -D_y \frac{\partial^2 w}{\partial \bar{y}^2}(x, \bar{b}), \quad (100)$$

and by substituting Eq. (98) into Eq. (100),

$$\left. \begin{aligned} \frac{\partial^2 w}{\partial \bar{y}^2}(x, \bar{b}) &= -\frac{M}{D_y} \\ &= \frac{\partial^2 \Phi_0(x, \bar{b})}{\partial \bar{y}^2} + \frac{\partial^2 \Phi_1(x, \bar{b})}{\partial \bar{y}^2} \alpha + \frac{\partial^2 \Phi_2(x, \bar{b})}{\partial \bar{y}^2} \alpha^2 + \dots \end{aligned} \right\} (101)$$

The governing equation for the first term of the solution is

$$D_x \nabla^4 \Phi_0(x, \bar{y}) = 0, \quad (102)$$

and the boundary conditions are:

$$\left. \begin{aligned} \Phi_0 &= 0 & \leadsto & \text{ALL EDGES} \\ \frac{\partial^2 \Phi_0}{\partial x^2} &= 0 & \leadsto & x = 0, a \\ \frac{\partial^2 \Phi_0}{\partial \bar{y}^2} &= 0 & \leadsto & y = 0 \\ \frac{\partial^2 \Phi_0}{\partial \bar{y}^2} &= -\frac{M}{D_y} & \leadsto & y = b. \end{aligned} \right\} (103)$$

The solution is taken in the form:

$$\Phi_0 = \Psi_0(x, \bar{y}) + f(\bar{y}) M, \quad (104)$$

where Ψ_0 satisfies homogeneous boundary conditions and

$$f(\bar{y}) = -\frac{1}{6D_y} \left[\frac{\bar{y}^3}{b} - b\bar{y} \right]. \quad (105)$$

From previous conditions, the function,

$$\begin{aligned} \Psi_0(x, \bar{y}) = \sum_{m=1}^{\infty} & \left[(C_1 + C_2 x) \cosh \lambda_m x \right. \\ & \left. + (C_3 + C_4 x) \sinh \lambda_m x \right] \sin \lambda_m \bar{y}, \end{aligned}$$

where the constants,

$$\begin{aligned} C_1 &= \frac{2M\bar{b}^2(-1)^m}{D_y m^3 \pi^3}, \\ C_2 &= -\frac{\lambda_m M\bar{b}^2(-1)^m}{D_y m^3 \pi^3} \frac{1 - \cosh \lambda_m a}{\sinh \lambda_m a}, \\ C_3 &= -\frac{M\bar{b}^2(-1)^m}{D_y m^3 \pi^3} \frac{(1 - \cosh \lambda_m a)(\lambda_m a - 2 \sinh \lambda_m a)}{\sinh^2 \lambda_m a}, \\ C_4 &= -\frac{\lambda_m M\bar{b}^2(-1)^m}{D_y m^3 \pi^3}, \end{aligned}$$

and

$$\lambda_m = \frac{m\pi}{b}.$$

The second term is taken in the form,

$$\Phi_1(x, y) = \sum_{m=1}^{\infty} \Theta_m(x) \sin \lambda_m y ,$$

where the functions $\Theta_m(x)$ are evaluated by:

$$\Theta_m(x) = (C_5 + C_6 x + A_0 x^2 + A_1 x^3) \cosh \lambda_m a + (C_7 + C_8 x + A_2 x^2 + A_3 x^3) \sinh \lambda_m a , \quad \left. \vphantom{\Theta_m(x)} \right\} (107)$$

where the constants,

$$C_5 = 0 ,$$

$$C_6 = 2A_0 a - \frac{3A_1 a}{\lambda_m} \left(\lambda_m a + \frac{\cosh \lambda_m a}{\sinh \lambda_m a} \right) - \frac{A_2}{\lambda_m} \left[1 + \frac{2\lambda_m a \cosh \lambda_m a}{\sinh \lambda_m a} - \frac{3A_3}{\lambda_m} \left(1 + \frac{\lambda_m a \cosh \lambda_m a}{\sinh \lambda_m a} \right) \right] ,$$

$$C_7 = \frac{A_0 a}{\lambda_m \sinh \lambda_m a} (\lambda_m a \cosh \lambda_m a + \sinh \lambda_m a) + \frac{A_1 a^2 \cosh \lambda_m a}{\lambda_m \sinh^2 \lambda_m a} (2\lambda_m a \sinh \lambda_m a + 3 \cosh \lambda_m a) + \frac{A_2 a}{\lambda_m \sinh^2 \lambda_m a} (\cosh \lambda_m a \sinh \lambda_m a + 2\lambda_m a + \lambda_m a \sinh^2 \lambda_m a) + \frac{A_3 a^2}{\lambda_m \sinh^2 \lambda_m a} (3 \cosh \lambda_m a \sinh \lambda_m a + 3\lambda_m a + 2\lambda_m a \sinh^2 \lambda_m a) ,$$

$$C_8 = - \frac{A_0}{\lambda_m} ,$$

and

$$A_0 = - \frac{(C_1 \lambda_m + C_4) \lambda_m}{8} ,$$

$$A_1 = - \frac{C_2 \lambda_m^2}{24} ,$$

$$A_2 = - \frac{(C_3 \lambda_m + C_6) \lambda_m}{8}$$

$$A_3 = - \frac{C_4 \lambda_m^2}{24} .$$

The evaluation of these terms is necessary before the deformation equation (98) can be solved.

A more general case of a non-uniform moment distributed along any line parallel to the x-axis (Fig. 15) is considered by Ando (Ref. 209).

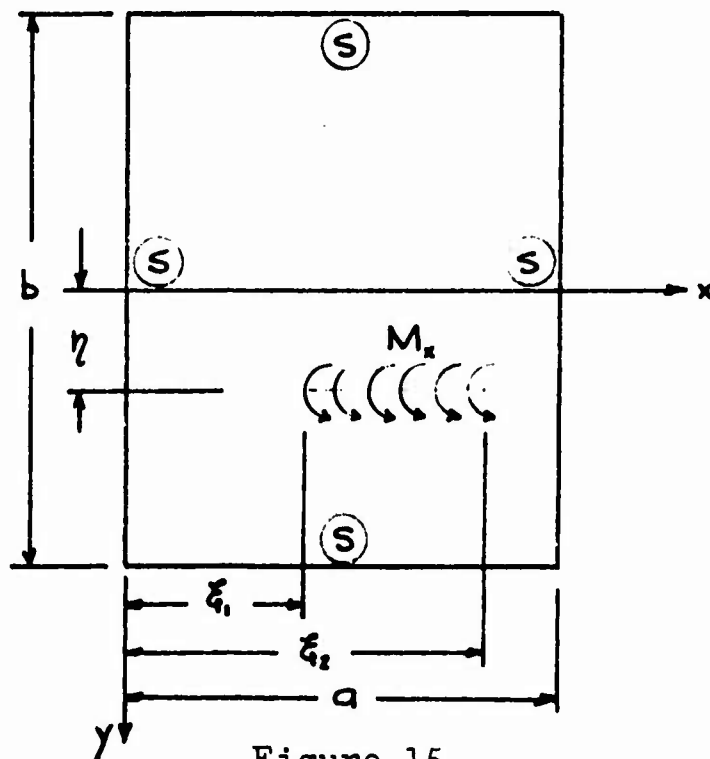


Figure 15
Simple Supports - Line Moment

The problem is solved by using the solution of Part 3.1.4 for a line loading. If a linear load, such as that of Figure 12, acts on the plate, and another load of the same intensity, parallel and very near to it is allowed to act simultaneously in the opposite direction, a line couple is created. This is shown in Figure 16.

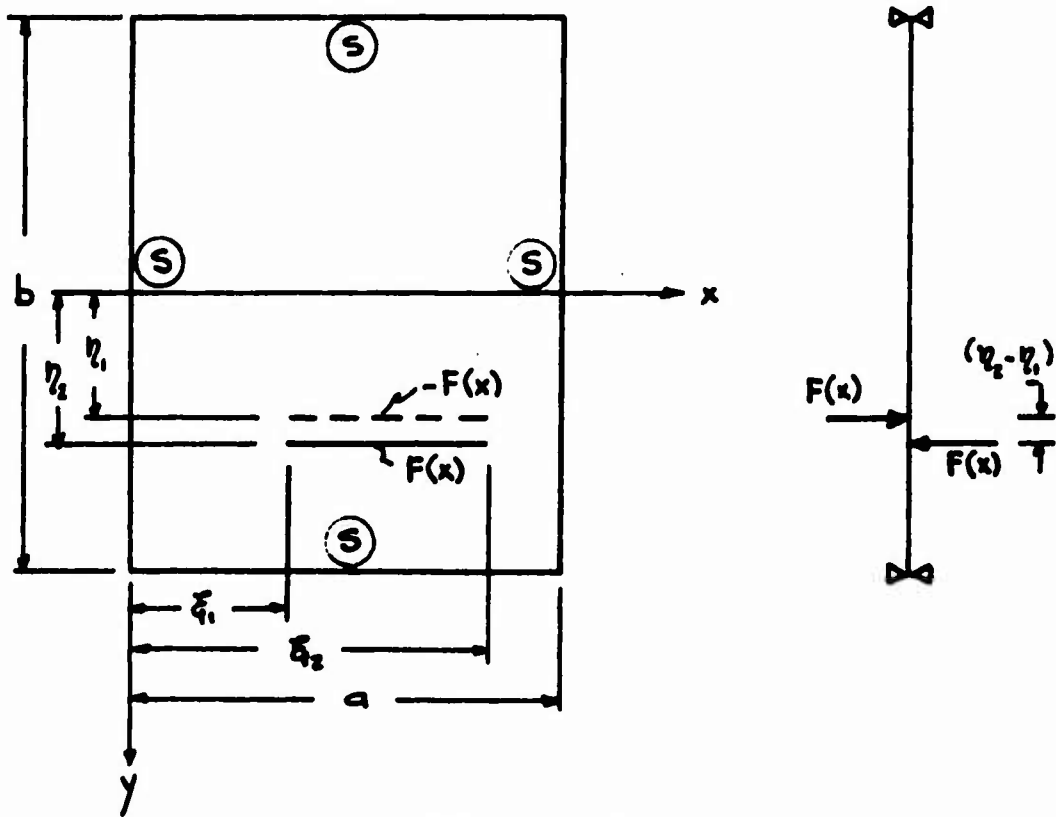


Figure 16
Simple Supports - Line-Load Couple

Thus, as

$$(\eta_2 - \eta_1) \rightarrow 0,$$

the loading of Figure 16 approaches that of Figure 15,
and

$$M(x) = \lim_{(\eta_2 - \eta_1) \rightarrow 0} F(x) (\eta_2 - \eta_1). \quad (108)$$

Thus, the deformation for two regions is again given
by Eqs. (89) and (90), and the constants for this
loading condition are:

$$C_1 = \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\cosh \frac{m\pi\lambda\eta}{a} - \left(\sinh \frac{m\pi\lambda\eta}{a} \right. \right. \\ \left. \left. \cdot \tanh \frac{m\pi\lambda y}{a} \right) \right],$$

$$C_2 = \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[\sinh \frac{m\pi\lambda\eta}{a} + \left(\cosh \frac{m\pi\lambda\eta}{a} \right. \right. \\ \left. \left. \cdot \coth \frac{m\pi\lambda y}{a} \right) \right],$$

$$C_3 = \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[\cosh \frac{m\pi\lambda\eta}{a} - \left(\sinh \frac{m\pi\lambda\eta}{a} \right. \right. \\ \left. \left. \cdot \tanh \frac{m\pi\lambda y}{a} \right) \right],$$

$$C_4 = \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\sinh \frac{m\pi\lambda\eta}{a} + \left(\cosh \frac{m\pi\lambda\eta}{a} \right. \right. \\ \left. \left. \cdot \coth \frac{m\pi\lambda y}{a} \right) \right],$$

and the loading term,

$$E_m = \frac{2a}{D_m \pi^2} \int_{\xi_1}^{\xi_2} M(\xi) \sin \frac{m\pi\xi}{a} d\xi .$$

The problem is greatly simplified, of course, when the line of the moment coincides with the x-axis. Then, the constants become:

$$\begin{aligned} C_1 &= - \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} = C_3 , \\ C_2 &= \frac{E_m \lambda^2 \bar{\lambda}^2}{2(\lambda^2 - \bar{\lambda}^2)} = -C_4 . \end{aligned}$$

3.2 - 2 Opposite Edges Clamped and the Other Two Simply Supported

Since the problem of an orthotropic rectangular plate having two opposite sides built in and the two remaining sides simply supported is of some practical importance, it deserves discussion here. Several loading conditions are considered and the method of Ando (Ref. 209) is utilized.

3.2.1 Uniformly Distributed Load

A plate with a uniformly distributed loading acting over the entire length, b , and over a portion of the length, a , is considered (see Figure 17). The notation (S) is again used to indicate simple supports, and (C) designates clamped edges.

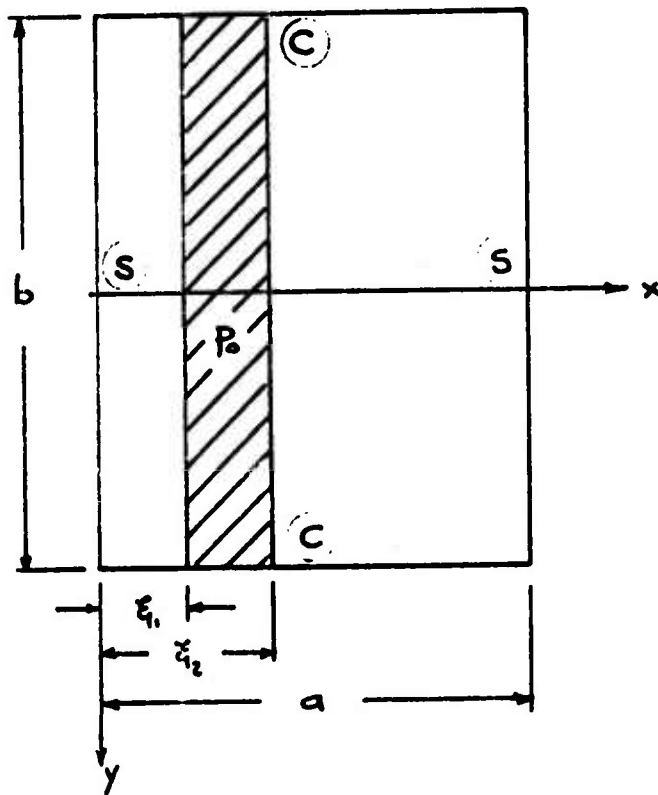


Figure 17
(S) - (C) - (S) - (C) - Uniform Load

The deformation equations (83), (84), and (85) are again used as the basic expressions. Due to the boundary conditions:

$$\left. \begin{aligned} w_2 = 0 = \frac{\partial w_2}{\partial y} \quad \text{at} \quad y = \frac{b}{2}, \\ w_3 = 0 = \frac{\partial w_3}{\partial y} \quad \text{at} \quad y = -\frac{b}{2}, \end{aligned} \right\} (109)$$

and because of the simplification of the loading (relative to that shown in Figure 7), the constants become:

$$\begin{aligned} C_1 = C_5 = C_9 &= \frac{A_m \bar{\lambda} \sinh \frac{m\pi \bar{\lambda} y}{a}}{\lambda \cosh \frac{m\pi \bar{\lambda} y}{a} \sinh \frac{m\pi \bar{\lambda} y}{a} - \bar{\lambda} \cosh \frac{m\pi \lambda y}{a} \sinh \frac{m\pi \lambda y}{a}}, \\ C_2 = C_4 = C_6 = C_8 = C_{10} = C_{12} &= 0, \\ C_3 = C_7 = C_{11} &= \frac{A_m \lambda \sinh \frac{m\pi \lambda y}{a}}{\bar{\lambda} \cosh \frac{m\pi \lambda y}{a} \sinh \frac{m\pi \bar{\lambda} y}{a} - \lambda \cosh \frac{m\pi \bar{\lambda} y}{a} \sinh \frac{m\pi \lambda y}{a}}. \end{aligned}$$

These values are substituted into Eqs. (83), (84), and (85) and deflections are found in any of the three regions. For the case of a uniform loading over the entire surface, $\xi_1 = 0$ and $\xi_2 = a$, in the evaluation of the load parameter,

$$\begin{aligned}
 A_m &= -\frac{2a^3 P_0}{D_x m^4 \pi^4} \int_{\xi_1}^{\xi_2} \sin \frac{m\pi \xi}{a} d\xi \\
 &= \frac{2a^4 P_0}{D_x m^5 \pi^5} \left(\cos \frac{m\pi \xi_1}{a} - \cos \frac{m\pi \xi_2}{a} \right).
 \end{aligned}$$

3.2.2 Concentrated Force

Next, the orthotropic rectangular plate of Figure 10 is considered. Here, however, the edge conditions are:

$$\begin{aligned}
 x = 0, a &\longrightarrow \text{simply supported,} \\
 y = \pm b/2 &\longrightarrow \text{clamped.}
 \end{aligned}$$

Eqs. (89) and (90) are used to describe the deformed condition, and the constants for these boundary conditions become:

$$\begin{aligned}
 C_1 &= \frac{P_0^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[-\bar{\lambda} \sinh \frac{m\pi \lambda \eta}{a} + \bar{\lambda} \gamma_1 \cosh \frac{m\pi \lambda \eta}{a} \right. \\
 &\quad \left. - \lambda \gamma_2 \cosh \frac{m\pi \lambda \eta}{a} \right] \sin \frac{m\pi \xi}{a},
 \end{aligned}$$

$$C_2 = \frac{Pa^2 \lambda \bar{\lambda}}{D_2 m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[\bar{\lambda} \cosh \frac{m\pi \lambda \eta}{a} + \bar{\lambda} \delta_1 \sinh \frac{m\pi \lambda \eta}{a} - \lambda \delta_2 \sinh \frac{m\pi \lambda \eta}{a} \right] \sin \frac{m\pi \xi}{a},$$

$$C_3 = \frac{Pa^2 \lambda \bar{\lambda}}{D_2 m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[\bar{\lambda} \sinh \frac{m\pi \lambda \eta}{a} + \bar{\lambda} \gamma_1 \cosh \frac{m\pi \lambda \eta}{a} - \lambda \gamma_2 \cosh \frac{m\pi \lambda \eta}{a} \right] \sin \frac{m\pi \xi}{a},$$

$$C_4 = \frac{Pa^2 \lambda \bar{\lambda}}{D_2 m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \left[-\bar{\lambda} \cosh \frac{m\pi \lambda \eta}{a} + \bar{\lambda} \delta_1 \sinh \frac{m\pi \lambda \eta}{a} - \lambda \delta_2 \sinh \frac{m\pi \lambda \eta}{a} \right] \sin \frac{m\pi \xi}{a},$$

where

$$\gamma_1 = \frac{\lambda \cosh \frac{m\pi \lambda \gamma}{a} \cosh \frac{m\pi \bar{\lambda} \gamma}{a} - \bar{\lambda} \sinh \frac{m\pi \lambda \gamma}{a} \sinh \frac{m\pi \bar{\lambda} \gamma}{a}}{\bar{\lambda} \cosh \frac{m\pi \lambda \gamma}{a} \sinh \frac{m\pi \bar{\lambda} \gamma}{a} - \lambda \cosh \frac{m\pi \bar{\lambda} \gamma}{a} \sinh \frac{m\pi \lambda \gamma}{a}},$$

$$\gamma_2 = \frac{\bar{\lambda}}{\bar{\lambda} \cosh \frac{m\pi \lambda \gamma}{a} \sinh \frac{m\pi \bar{\lambda} \gamma}{a} - \lambda \cosh \frac{m\pi \bar{\lambda} \gamma}{a} \sinh \frac{m\pi \lambda \gamma}{a}},$$

$$\delta_1 = \frac{\bar{\lambda} \cosh \frac{m\pi \lambda \gamma}{a} \cosh \frac{m\pi \bar{\lambda} \gamma}{a} - \lambda \sinh \frac{m\pi \lambda \gamma}{a} \sinh \frac{m\pi \bar{\lambda} \gamma}{a}}{\bar{\lambda} \cosh \frac{m\pi \bar{\lambda} \gamma}{a} \sinh \frac{m\pi \lambda \gamma}{a} - \lambda \cosh \frac{m\pi \lambda \gamma}{a} \sinh \frac{m\pi \bar{\lambda} \gamma}{a}},$$

$$\delta_2 = \frac{\bar{\lambda}}{\bar{\lambda} \cosh \frac{m\pi \bar{\lambda} \gamma}{a} \sinh \frac{m\pi \lambda \gamma}{a} - \lambda \cosh \frac{m\pi \lambda \gamma}{a} \sinh \frac{m\pi \bar{\lambda} \gamma}{a}}.$$

In the special case of the load P acting on the x -axis,

$$\eta = 0 ,$$

and the constants become:

$$C_1 = \frac{Pa^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} (\bar{\lambda} \gamma_1 - \lambda \gamma_2) \sin \frac{m\pi \xi}{a} = C_3 ,$$

$$C_2 = \frac{Pa^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} \sin \frac{m\pi \xi}{a} = -C_4 .$$

When the concentrated load is located at the center of the plate,

$$\eta = 0 , \quad \xi = \frac{a}{2} ,$$

and the constants are simplified further:

$$C_1 = \frac{(-1)^{\frac{m-1}{2}} Pa^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} (\bar{\lambda} \gamma_1 - \lambda \gamma_2) = C_3 ,$$

$$C_2 = \frac{(-1)^{\frac{m-1}{2}} Pa^2 \lambda \bar{\lambda}}{D_x m^3 \pi^3 (\lambda^2 - \bar{\lambda}^2)} = -C_4 .$$

$$(m = 1, 3, 5, \dots)$$

3.2.3 Line Loading

The plate illustrated in Figure 12 is considered next. The edge conditions are:

$$\begin{array}{ll} x = 0, a & \longrightarrow \text{ simply supported,} \\ y = \pm b/2 & \longrightarrow \text{ clamped.} \end{array}$$

The deformation equations (89) and (90) are again used to give the forms of the deflected surface for the two regions of y . For the boundary conditions considered here, the constants become:

$$C_1 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\bar{\lambda} \sinh \frac{m\pi\lambda\eta}{a} + \bar{\lambda} \gamma_1 \cosh \frac{m\pi\lambda\eta}{a} - \lambda \gamma_2 \cosh \frac{m\pi\lambda\eta}{a} \right],$$

$$C_2 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} \left[\bar{\lambda} \cosh \frac{m\pi\lambda\eta}{a} + \bar{\lambda} \delta_1 \sinh \frac{m\pi\lambda\eta}{a} - \lambda \delta_2 \sinh \frac{m\pi\lambda\eta}{a} \right],$$

$$C_3 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} \left[\bar{\lambda} \sinh \frac{m\pi\lambda\eta}{a} + \bar{\lambda} \gamma_1 \cosh \frac{m\pi\lambda\eta}{a} - \lambda \gamma_2 \cosh \frac{m\pi\lambda\eta}{a} \right],$$

$$C_4 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} \left[-\bar{\lambda} \cosh \frac{m\pi\lambda\eta}{a} + \bar{\lambda} \delta_1 \sinh \frac{m\pi\lambda\eta}{a} - \lambda \delta_2 \sinh \frac{m\pi\lambda\eta}{a} \right],$$

where f_m is defined in Paragraph 3.1.4 and the parameters γ and δ are given in the preceding paragraph.

For the particular case of the line load F being applied along the x -axis,

$$\eta = 0,$$

and the constants simplify to:

$$C_1 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} (\bar{\lambda} \gamma_1 - \lambda \gamma_2) = C_3,$$

$$C_2 = \frac{f_m \lambda \bar{\lambda}}{2(\lambda^2 - \bar{\lambda}^2)} = -C_4.$$

3.3 Infinitely Long Plate

An infinitely long plate with the long sides simply supported is now considered. Since the ratio of b/a is very large, the use of the single series, Lévy-type, solution is justified and the deformation is taken in the form of Eq. (49):

$$w = \sum_m Y_m \sin \frac{m\pi x}{a},$$

and Y_m satisfies the condition:

$$D_y Y_m'' - 2D_y \frac{m^2 \pi^2}{a^2} Y_m + D_x \frac{m^4 \pi^4}{a^4} Y_m = 0. \quad (110)$$

The roots of the characteristic equation have four forms:

$$\text{roots} = \pm \frac{m\pi}{a} \sqrt{\frac{D_{xy}}{D_y} \pm \sqrt{\frac{D_{xy}^2}{D_y^2} - \frac{D_x}{D_y}}}. \quad (111)$$

Using the parameters,

$$\lambda = \sqrt[4]{\frac{D_y}{D_x}}, \quad \mu = \frac{D_{xy}}{\sqrt{D_x D_y}},$$

three cases (as discussed in Paragraph 3.1.1) must be considered:

$$\text{Case I : } \mu > 1 \quad \Rightarrow \left(\frac{D_{xy}}{D_x} \right)^2 > \frac{D_y}{D_x},$$

$$\text{Case II : } \mu = 1 \quad \Rightarrow \left(\frac{D_{xy}}{D_x} \right)^2 = \frac{D_y}{D_x},$$

$$\text{Case III: } \mu < 1 \quad \Rightarrow \left(\frac{D_{xy}}{D_x} \right)^2 < \frac{D_y}{D_x}.$$

3.3.1 Line Loading

Let the line load, q , act along the x -axis of an infinitely long plate, as shown in Figure 18. The deformation will be taken in the single series form, as shown by Timoshenko and Woinowsky-Krieger (Ref. 97).

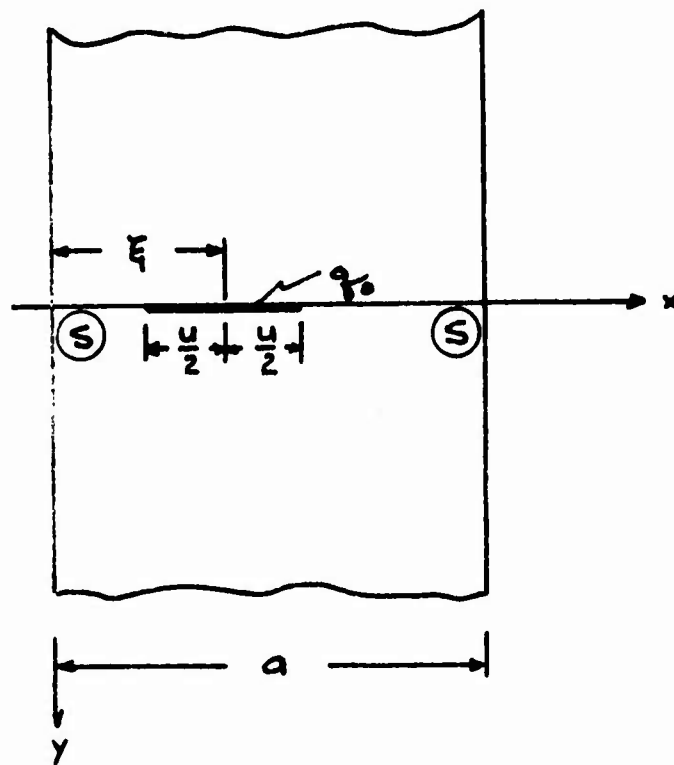


Figure 18
Infinite Plate - Line Load

For Case I, all the roots of Eq. (111) are real and the deformation,

$$w_z = \frac{q_0 a^4}{2\pi^4 D_x (\alpha^2 - \beta^2)} \sum_m \frac{1}{m^3} (\alpha e^{-\frac{my}{\alpha}} - \beta e^{-\frac{my}{\beta}}) \sin \frac{m\pi x}{a}. \quad (112)$$

For Case II, there are two double roots, and the deformation,

$$w_{II} = \frac{q_0 a^3}{\pi^4 D_x} \sum_m \frac{1}{m^4} \left(1 + \frac{m\pi y}{a}\right) e^{-\frac{m\pi y}{a}} \sin \frac{m\pi x}{a}. \quad (113)$$

In the third case, the deformation has the form,

$$w_{III} = \frac{q_0 a^2}{4\pi^2 \sqrt{D_x D_y}} \sum_m \frac{1}{m^3} \left(\alpha' \sin \frac{my}{\alpha'} + \beta' \cos \frac{my}{\alpha'}\right) e^{-\frac{my}{\beta'}} \sin \frac{m\pi x}{a}. \quad (114)$$

The following notation was introduced in the above expressions for the sake of brevity:

$$\alpha = \frac{a\lambda}{\pi} \sqrt{\mu + \sqrt{\mu^2 - 1}},$$

$$\beta = \frac{a\lambda}{\pi} \sqrt{\mu - \sqrt{\mu^2 - 1}},$$

$$\alpha' = \frac{a\lambda}{\pi} \sqrt{\frac{2}{1-\mu}},$$

$$\beta' = \frac{a\lambda}{\pi} \sqrt{\frac{2}{1+\mu}},$$

and q_0 is the load per unit length.

3.3.2 Concentrated Force

Now, a concentrated load, P , is applied at $(x = \xi, y = 0)$ of Figure 18. Then, the deformation expressions for the three cases of elastic properties become:

$$w_I = \frac{Pa^3}{\pi^4 D_x (\alpha^2 - \beta^2)} \sum_m \frac{1}{m^3} (\alpha e^{-\frac{m\pi y}{a}} - \beta e^{-\frac{m\pi y}{b}}) \sin \frac{m\pi \xi}{a} \sin \frac{m\pi x}{a}, \quad (115)$$

$$w_{II} = \frac{Pa^2}{2\pi^3 \lambda D_x} \sum_m \frac{1}{m^3} \left(1 + \frac{m\pi y}{a\lambda}\right) e^{-\frac{m\pi y}{\lambda}} \sin \frac{m\pi \xi}{a} \sin \frac{m\pi x}{a}, \quad (116)$$

$$w_{III} = \frac{Pa}{2\pi^2 \sqrt{D_x D_y}} \sum_m \frac{1}{m^3} \left(\alpha' \sin \frac{m\pi y}{\alpha'} + \beta' \cos \frac{m\pi y}{\alpha'} \right) e^{-\frac{m\pi y}{\alpha'}} \sin \frac{m\pi \xi}{a} \sin \frac{m\pi x}{a}. \quad (117)$$

Nowacki, in Reference 71, obtains a solution in closed form for the bending moments due to a load of this type.

3.4 All Edges Clamped

The difficulty presented by the use of fixed supports in the theory of plates is many times greater than that encountered in beam theory. The usual method of solution is superposition. Generally, the deformation is found for a simply supported plate under the actual

loading, and it is added to the deformation due to distributed moments along the edges.

Any of the previously discussed techniques can be applied to solving this problem by superposition. Since Ando's method has been used rather extensively in this chapter, it will again be applied here.

The problem is divided into two parts: 1.) Simple supports at $x = \pm a/2$ edges, fixed supports at $y = \pm b/2$ edges, and under actual loading (Fig. 19a); 2.) Fixed supports on $y = \pm b/2$ edges and moment distributed along $x = \pm a/2$ edges (Fig. 19b). The combined effect of the two parts is the clamped plate shown in Figure 19c.

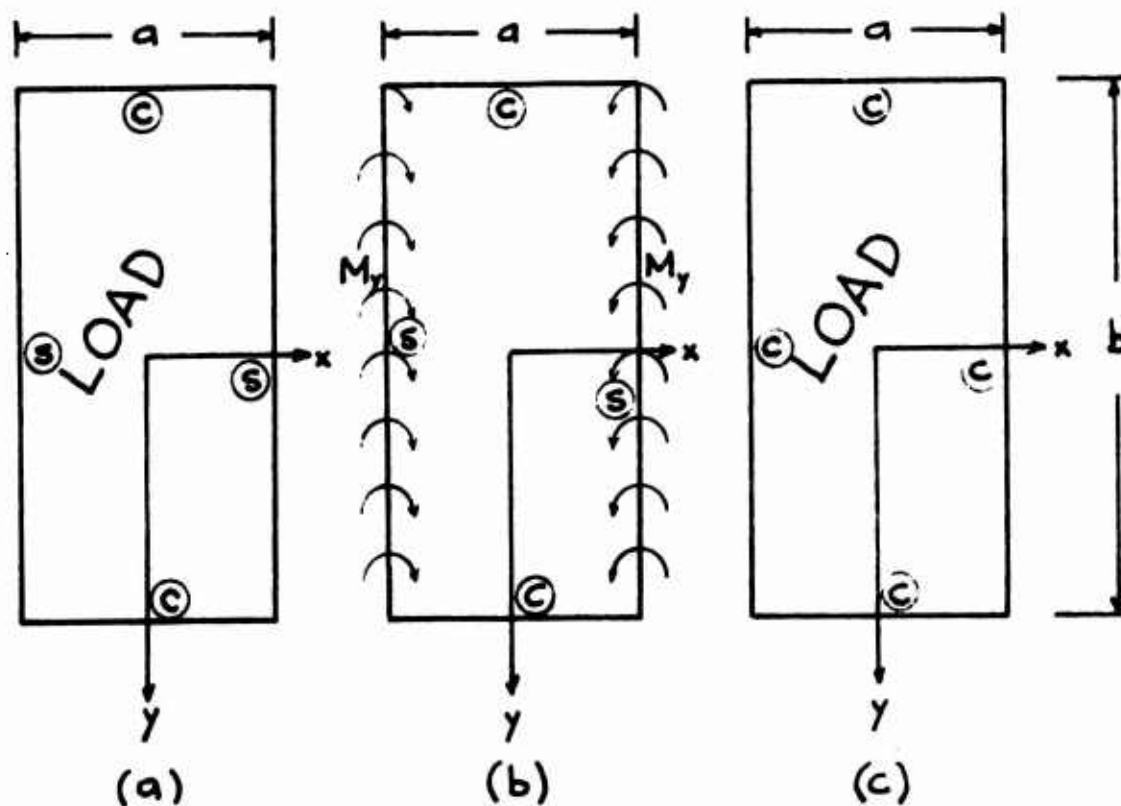


Figure 19
Clamped Plate by Superposition

The total deformation, w , is composed of two terms corresponding to the above breakdown:

$$w = w_a + w_b, \quad (118)$$

where

w_a = deflection of plate of Figure 19a under actual loading,

w_b = deflection of plate of Figure 19b.

The edge moment expression is taken as:

$$M_y = \sum_m E_m \sin \frac{m\pi y}{b}, \quad (119)$$

where E_m defines its intensity.

From this, and from previous considerations, the deformations:

$$w_a = \sum_m \left[(-1)^{\frac{m-1}{2}} (A_m + C_1 \cosh \frac{m\pi\lambda y}{a} + C_2 \cosh \frac{m\pi\bar{\lambda} y}{a}) \cdot \cos \frac{m\pi x}{a} \right], \quad (120)$$

$$w_b = \sum_m \left[(C_3 \cosh \frac{m\pi\lambda y}{a} + C_4 \cosh \frac{m\pi\bar{\lambda} y}{a}) \cos \frac{m\pi x}{a} + (C_5 \cosh \frac{m\pi\lambda x}{a} + C_6 \cosh \frac{m\pi\bar{\lambda} x}{a}) \cos \frac{m\pi y}{b} \right]. \quad (121)$$

Evaluating boundary conditions, the total deformation becomes:

$$w = \sum_m \left\{ (-1)^{\frac{m-1}{2}} A_m + [(-1)^{\frac{m-1}{2}} C_1 + C_3] \cosh \frac{m\pi\lambda y}{a} + [(-1)^{\frac{m-1}{2}} C_2 + C_4] \cosh \frac{m\pi\bar{\lambda} y}{a} \right\} \cos \frac{m\pi x}{a} + \sum_m (C_5 \cosh \frac{m\pi\lambda x}{a} + C_6 \cosh \frac{m\pi\bar{\lambda} x}{a}) \cos \frac{m\pi y}{b}. \quad (122)$$

The constants of Eq. (122) have the following values:

$$\left. \begin{aligned} C_1 &= - \frac{A_m \bar{\lambda} \sinh \frac{m\pi\bar{\lambda} y}{a}}{\bar{\lambda} \cosh \frac{m\pi\lambda y}{a} \sinh \frac{m\pi\bar{\lambda} y}{a} - \lambda \cosh \frac{m\pi\bar{\lambda} y}{a} \sinh \frac{m\pi\lambda y}{a}}, \\ C_2 &= - \frac{A_m \lambda \sinh \frac{m\pi\lambda y}{a}}{\lambda \cosh \frac{m\pi\bar{\lambda} y}{a} \sinh \frac{m\pi\lambda y}{a} - \bar{\lambda} \cosh \frac{m\pi\lambda y}{a} \sinh \frac{m\pi\bar{\lambda} y}{a}}, \\ C_3 &= \frac{4ab \cosh \frac{m\pi\bar{\lambda} y}{a}}{\lambda \cosh \frac{m\pi\lambda y}{a} \sinh \frac{m\pi\bar{\lambda} y}{a} - \bar{\lambda} \cosh \frac{m\pi\bar{\lambda} y}{a} \sinh \frac{m\pi\lambda y}{a}} \sum_n \left\{ \left[\frac{(-1)^{\frac{n-1}{2}} (-1)^{\frac{m-1}{2}} E_m}{D_n n^4 \pi^2} \right] \right. \\ &\quad \left. \cdot \left[\frac{1}{(\lambda^2 - \bar{\lambda}^2) \left[\left(\frac{a}{b} \right)^2 \lambda^2 + \left(\frac{m}{n} \right)^2 \right]} + \frac{1}{(\lambda^2 - \bar{\lambda}^2) \left[\left(\frac{a}{b} \right)^2 \bar{\lambda}^2 + \left(\frac{m}{n} \right)^2 \right]} \right] \right\}, \\ C_4 &= \frac{4ab \cosh \frac{m\pi\lambda y}{a}}{\lambda \cosh \frac{m\pi\bar{\lambda} y}{a} \sinh \frac{m\pi\lambda y}{a} - \bar{\lambda} \cosh \frac{m\pi\lambda y}{a} \sinh \frac{m\pi\bar{\lambda} y}{a}} \sum_n \left\{ \left[\frac{(-1)^{\frac{n-1}{2}} (-1)^{\frac{m-1}{2}} E_m}{D_n n^4 \pi^2} \right] \right. \\ &\quad \left. \cdot \left[\frac{1}{(\lambda^2 - \bar{\lambda}^2) \left[\left(\frac{a}{b} \right)^2 \lambda^2 + \left(\frac{m}{n} \right)^2 \right]} + \frac{1}{(\lambda^2 - \bar{\lambda}^2) \left[\left(\frac{a}{b} \right)^2 \bar{\lambda}^2 + \left(\frac{m}{n} \right)^2 \right]} \right] \right\}, \\ C_5 &= \frac{b^2 E_m}{D_n m^2 \pi^2 (\lambda^2 - \bar{\lambda}^2) \cosh \frac{m\pi\lambda y}{a}}, \\ C_6 &= \frac{b^2 E_m}{D_n m^2 \pi^2 (\lambda^2 - \bar{\lambda}^2) \cosh \frac{m\pi\bar{\lambda} y}{a}}. \end{aligned} \right\} \quad (123)$$

The constant, A_m , is evaluated for each particular loading condition:

a.) Uniformly distributed load (p_0):

$$A_m = \frac{4 p_0 a^4}{D_x m^5 \pi^5} \quad (124)$$

b.) Concentrated load (P - at distance ξ from left y-edge):

$$A_m = \frac{2 P a^3}{D_x m^4 \pi^4} \sin \frac{m \pi \xi}{a} \quad (125)$$

c.) Hydrostatic pressure (maximum p_0 - varies along x-axis):

$$A_m = \frac{2 (-1)^{m+1} p_0 a^4}{D_x m^5 \pi^5} \quad (126)$$

3.5 Summary

Most of the available solutions to the problem of bending of rectangular orthotropic plates require the calculation of the deformation, w , as an intermediate step prior to the computation of desired stresses. Regardless of the method used to arrive at the deformation function, the expressions given in Part 1 are applied in the determination of stresses, forces, and moments.

According to Eqs. (27) and (28), the vertical shears,

$$Q_x = -D_x \left[\frac{\partial^3 w}{\partial x^3} + \mu_{xy} \frac{\partial^3 w}{\partial x \partial y^2} \right] - D_r \frac{\partial^3 w}{\partial x \partial y^2},$$

$$Q_y = -D_y \left[\frac{\partial^3 w}{\partial y^3} + \mu_{yx} \frac{\partial^3 w}{\partial x^2 \partial y} \right] - D_r \frac{\partial^3 w}{\partial x^2 \partial y}.$$

The moments, from Eqs. (14), (15), and (16) are given by:

$$M_x = -\frac{t^3}{12(s_{11}s_{22} - s_{12}^2)} \left[s_{22} \frac{\partial^2 w}{\partial x^2} - s_{12} \frac{\partial^2 w}{\partial y^2} \right],$$

$$M_y = -\frac{t^3}{12(s_{11}s_{22} - s_{12}^2)} \left[s_{11} \frac{\partial^2 w}{\partial y^2} - s_{12} \frac{\partial^2 w}{\partial x^2} \right],$$

$$M_{xy} = \frac{t^3}{6s_{66}} \frac{\partial^2 w}{\partial x \partial y}.$$

The bending and shear stresses, respectively, are given by Eqs. (11), (12), and (13) as:

$$\sigma_x = -\frac{z}{s_{11}s_{22} - s_{12}^2} \left[s_{22} \frac{\partial^2 w}{\partial x^2} - s_{12} \frac{\partial^2 w}{\partial y^2} \right],$$

$$\sigma_y = -\frac{z}{s_{11}s_{22} - s_{12}^2} \left[s_{11} \frac{\partial^2 w}{\partial y^2} - s_{12} \frac{\partial^2 w}{\partial x^2} \right],$$

$$\tau_{xy} = \frac{2z}{s_{66}} \frac{\partial^2 w}{\partial x \partial y}.$$

The meanings of the parameters used in the above expressions, and the coordinate system in which they are oriented, are given in Part 1 of this report.

PART 4 - ELASTIC STABILITY

A great deal of research has been devoted to the problem of the stability of various types of stiffened plates, with reliance upon orthotropic plate theory. More progress has been made in buckling theory than in bending theory for two reasons: First, the buckling problem of aircraft wings has been of greater interest than the bending problem, and, second, the solution of the former is considerably simpler than that of transverse bending.

Some of the authors whose work is of importance due to either original contribution or summary of available methods in the general area of buckling of orthotropic plates are the following: Barg (Ref. 5), Gerard (Ref. 235), Heck and Ebner (Ref. 30), Hu (Ref. 36), Huber (Ref. 37), Kollbrunner and Meister (Ref. 53), Lekhnitski (Ref. 58), Nowacki (Ref. 71), Pflüger (Ref. 73), Pochop (Ref. 266), Radok (Ref. 267), Sawczuk (Ref. 82), Schmit and Kicher (Ref. 317), Schultz (Ref. 83), Seide (Ref. 280), Seydel (Refs. 84 and 282), Shuleshko (Ref. 86), Sokolowski (Ref. 284), Strasser (Ref. 285), Thielemann (Ref. 95), Timoshenko (Ref. 289), Timoshenko and Gere (Ref. 96), Wilde (Ref. 105), Wittrick (Ref. 107), and Yusuff (Refs. 111 and 300).

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Among the contributors to special problems are: Feldman (Ref. 232), who presents a study of square orthotropic plates; Caldwell (Ref. 214), who considers the special case of a plate in edge compression with reactions provided by edge shear; Rockey and Cook (Refs. 269 and 270) and Symonds (Ref. 287), who solve the shear buckling problem; Floor and Burgerhout (Ref. 233), Hayashi (Ref. 27), and Wang and Zuckerberg (Ref. 294), all of whom investigate the post-buckling behavior of orthotropic plates; Klitchieff (Ref. 250), who limits his study to plates with longitudinal reinforcement.

The contributors to the buckling of sandwich plates are not mentioned here. A summary of their work is contained in the chapter concerning the behavior of sandwich construction.

This chapter contains only the orthotropic buckling theory. It should be noted immediately that this theory cannot be applied indiscriminately to all stiffened plates. For this reason, limitations are discussed first.

4.1 Limitations

Since orthotropic plate theory is based upon average flexural and twisting rigidities (Eqs. (20), (21), (22)), it can be stated, in general, that the theory can be safely applied to any plate with a large number of equal and equidistant stiffeners parallel to one of the edges (Ref. 96).

In addition, Gerard and Becker (Ref. 236) show by test results that orthotropic theory may be used in the following cases: a.) plate in compression with three or more stiffeners; b.) plate in shear with any number of longitudinal stiffeners; c.) any plate with transverse stiffeners which has a low or high value of

$\frac{EI}{bD}$,
where

EI = stiffness of stiffener,

b = width of plate,

D = rigidity of plate.

4.2 General Equation

With reference to the derivation of Eq. (31), if the transverse load, p , is now replaced by in-plane forces, N_x , N_y , and N_{xy} , the differential equation of

the deformed plate is written:

$$\left. \begin{aligned} D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} \\ + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0. \end{aligned} \right\} (127)$$

Or, in terms of the stresses, the last three terms of Eq. (127) can be written:

$$\dots + t \left[\sigma_x \frac{\partial^2 w}{\partial x^2} + 2\tau_{xy} \frac{\partial^2 w}{\partial x \partial y} + \sigma_y \frac{\partial^2 w}{\partial y^2} \right], \quad (128)$$

where t is the thickness of the orthotropic plate, and the rigidities are defined by Eqs. (20), (21), (22), and (32).

Buckling due to the compressive loads per unit length, N_x and N_y , due to shear per unit length, N_{xy} , and due to combined loadings will be considered here.

4.3 Simple Supports

The orthotropic plate with simple supports along all edges (Fig. 20), is first considered. The following paragraphs give solutions for various loading conditions.

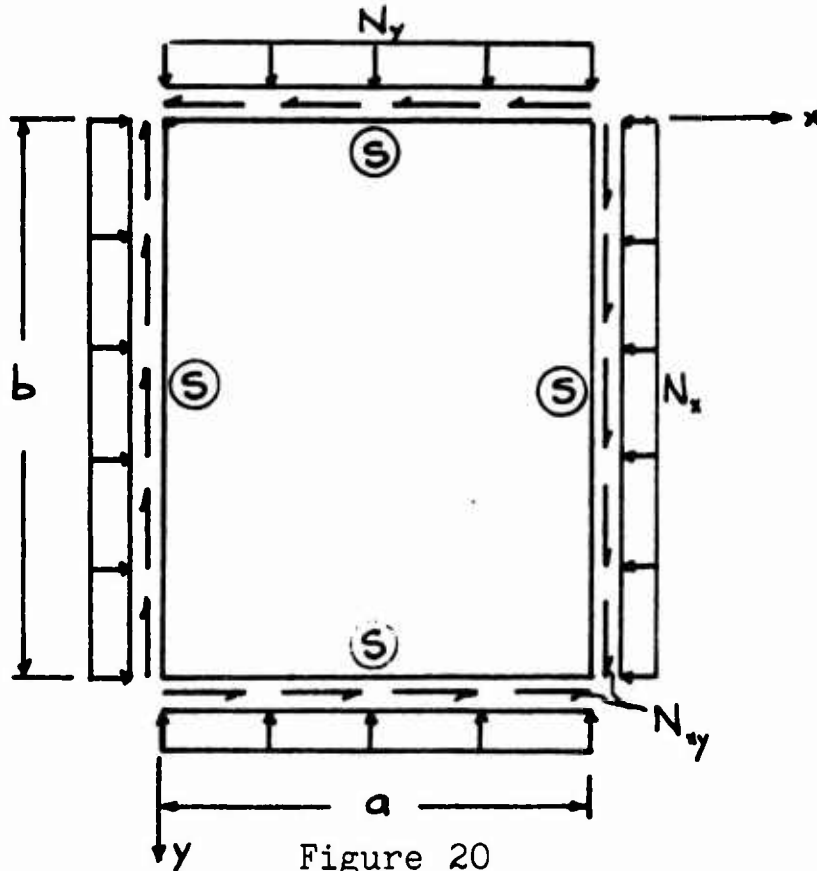


Figure 20
Simply Supported Plate

4.3.1 Uniaxial Compression

Let the plate of Figure 20 be loaded uniaxially by the compressive load N_x alone. Then, Eq. (128) becomes:

$$D \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + t\sigma_x \frac{\partial^2 w}{\partial x^2} = 0. \quad (129)$$

The deflected surface can be taken in the form of a double series,

$$w = \sum_m \sum_n w_0 \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} . \quad (130)$$

But, it is known that the critical condition exists when $m = 1$, $n = 1$, and the deformation,

$$w = w_0 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} . \quad (131)$$

Substituting this into Eq. (129) and solving for the stress,

$$\sigma_x = \frac{\pi^2}{b^2 t} \left[D_x \frac{b^2}{a^2} + 2D_{xy} + D_y \frac{a^2}{b^2} \right] . \quad (132)$$

The smallest value for the critical stress is obtained when

$$\frac{a}{b} = \sqrt{\frac{D_x}{D_y}} , \quad (133)$$

and the critical buckling stress, as given by Timoshenko and Gere (Ref. 96),

$$\sigma_{cr} = \frac{2\pi^2}{b^2 t} \left[\sqrt{D_x D_y} + D_{xy} \right] \quad (134)$$

Lundquist and Stowell (Ref. 253) give the buckling stress, for the condition of Eq. (133), as

$$\sigma_{cr} = \frac{\pi^2 t^2}{12 b^2} \left[2\sqrt{D_x D_y} + \mu_{xy} D_x + \mu_{yx} D_y + 4G \right] . \quad (135)$$

Wittrick, in Reference 107, gives an explicit solution to the buckling problem. Using Eq. (130) as the equation of the deformed plate, he takes as the solution,

$$k = \left[\frac{(a/b)_e}{m} + \frac{m}{(a/b)_e} \right]^2, \quad (136)$$

where

$$k = \text{buckling coefficient,}$$

$$(a/b)_e = \text{effective aspect ratio.}$$

The buckling coefficient is of the same type as that used for isotropic plates and can be applied if the effective aspect ratio,

$$\left(\frac{a}{b}\right)_e = \frac{a}{b} \sqrt[4]{\frac{D_y}{D_x}}, \quad (137)$$

is employed.

A plot of Eq. (136) is given in Figure 21. From this, the coefficient, k , is evaluated for a given value of $(a/b)_e$. This quantity is then substituted into the expression for the critical buckling load per unit length,

$$(N_x)_{cr} = \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \left[k - 2 + \frac{D_x}{\sqrt{D_x D_y}} \right] \quad (138)$$

4.3.2 Biaxial Compression

The plate of Figure 20 is now loaded by compressive loads in both directions. Thus, Eq. (127) becomes:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \quad (139)$$

For this loading condition, Wittrick (Ref. 107) applies the lower curve of Figure 21, but the effective aspect ratio must be modified to take into account the load N_y . Thus, the effective aspect ratio used for this case,

$$\left(\frac{a}{b}\right)_e = \frac{a}{b} \left[\frac{D_y}{D_x} \left(1 - \frac{b^2 N_y}{\pi^2 D_y} \right) \right]^{\frac{1}{4}}, \quad (140)$$

is first evaluated. The buckling coefficient, k , is found from Figure 21, and the following expression is used to complete the analysis:

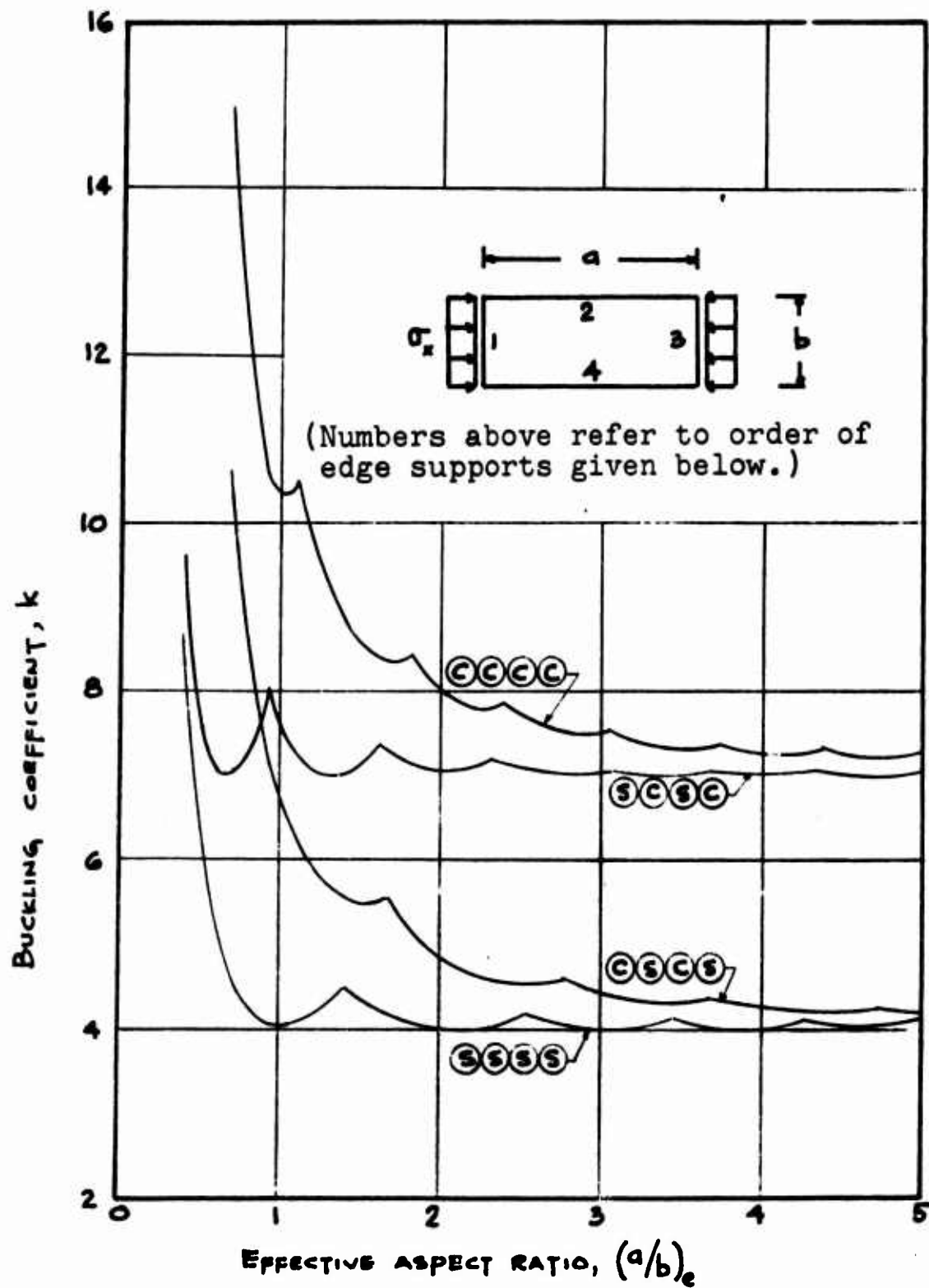


Figure 21
Buckling Coefficients for Uniaxially Loaded Plates
(from Reference 107)

$$k = 2 + \frac{(b^2 N_x / \pi^2) - D_x}{\sqrt{D_x D_y} (1 - \frac{b^2 N_y}{\pi^2 D_y})} \quad (141)$$

4.3.3 Edge Shear

When the plate shown in Figure 20 is acted upon by edge shears alone, the basic equation (127) becomes:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} = 0. \quad (142)$$

Seydel (Ref. 84) also makes use of the isotropic-type buckling coefficient to give the expression for the critical shear force per unit length,

$$(N_{xy})_{cr} = k_s \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \quad (143)$$

The shear buckling coefficient, k_s , is dependent upon the effective aspect ratio of Eq. (137) and upon the parameter $D_x / 2 \sqrt{D_x D_y}$. The coefficient is obtained from Figure 22a (which is given by Gerard and Becker in Reference 236), and it is substituted into Eq. (143).

A similar result is given by Heck and Ebner (Ref. 30) in the form:

$$(N_{xy})_{cr} = C_s \frac{\sqrt{D_x D_y}}{\left(\frac{b}{2}\right)^2} , \quad (144)$$

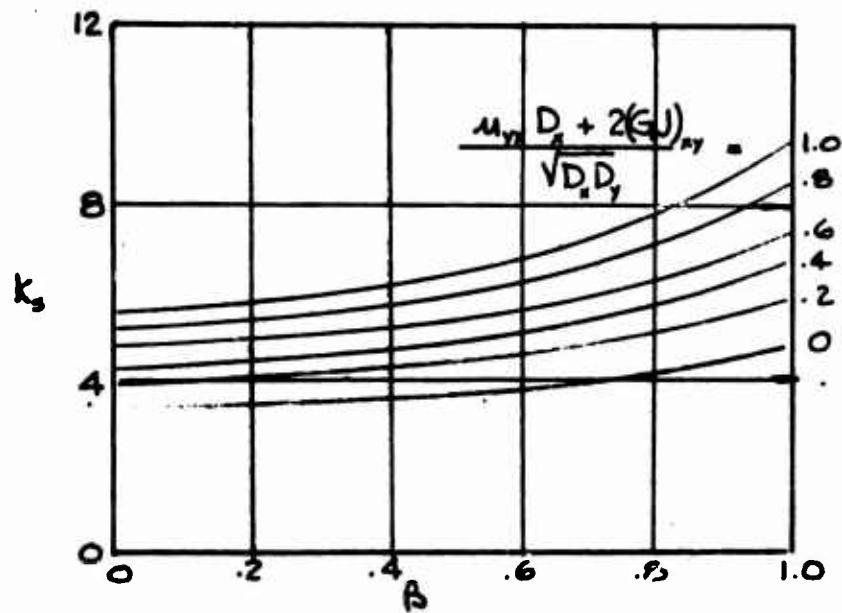
where C_s is a coefficient dependent on the parameters,

$$\theta = \frac{\sqrt{D_x D_y}}{D_{xy}} , \quad \beta = \frac{1}{(a/b)_c} = \frac{b}{a} \sqrt{\frac{D_x}{D_y}} . \quad (145)$$

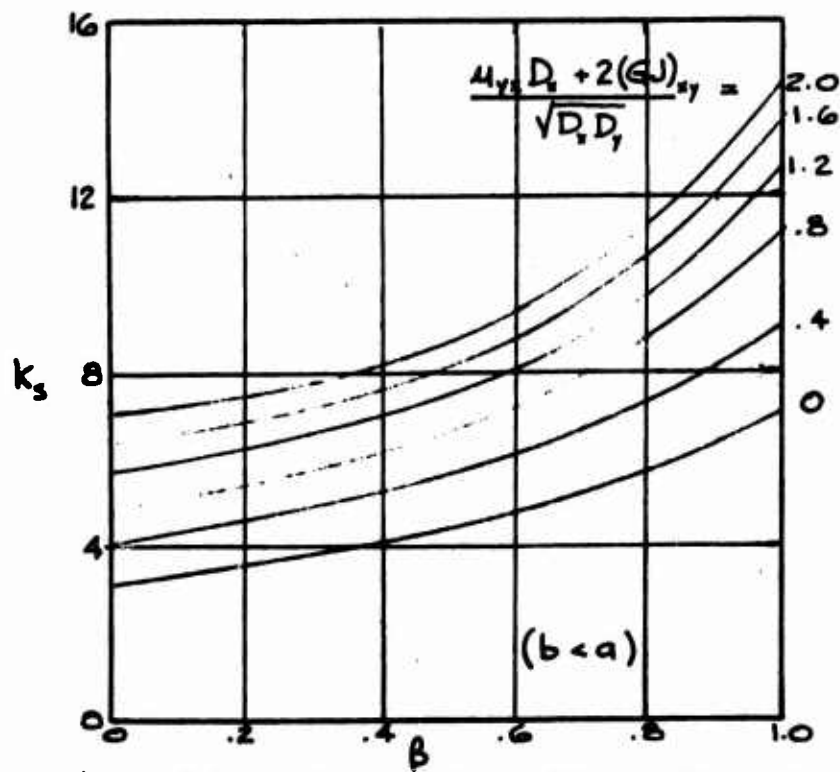
For ranges

$$0 \leq \beta \leq 1 , \quad \theta \geq 1 ,$$

the curves in Figure 23 apply, and C_s is found and substituted into Eq. (144).



a.) Simply Supported Edges



b.) x-Edges Clamped, y-Edges Hinged

Figure 22
Shear Buckling Coefficients

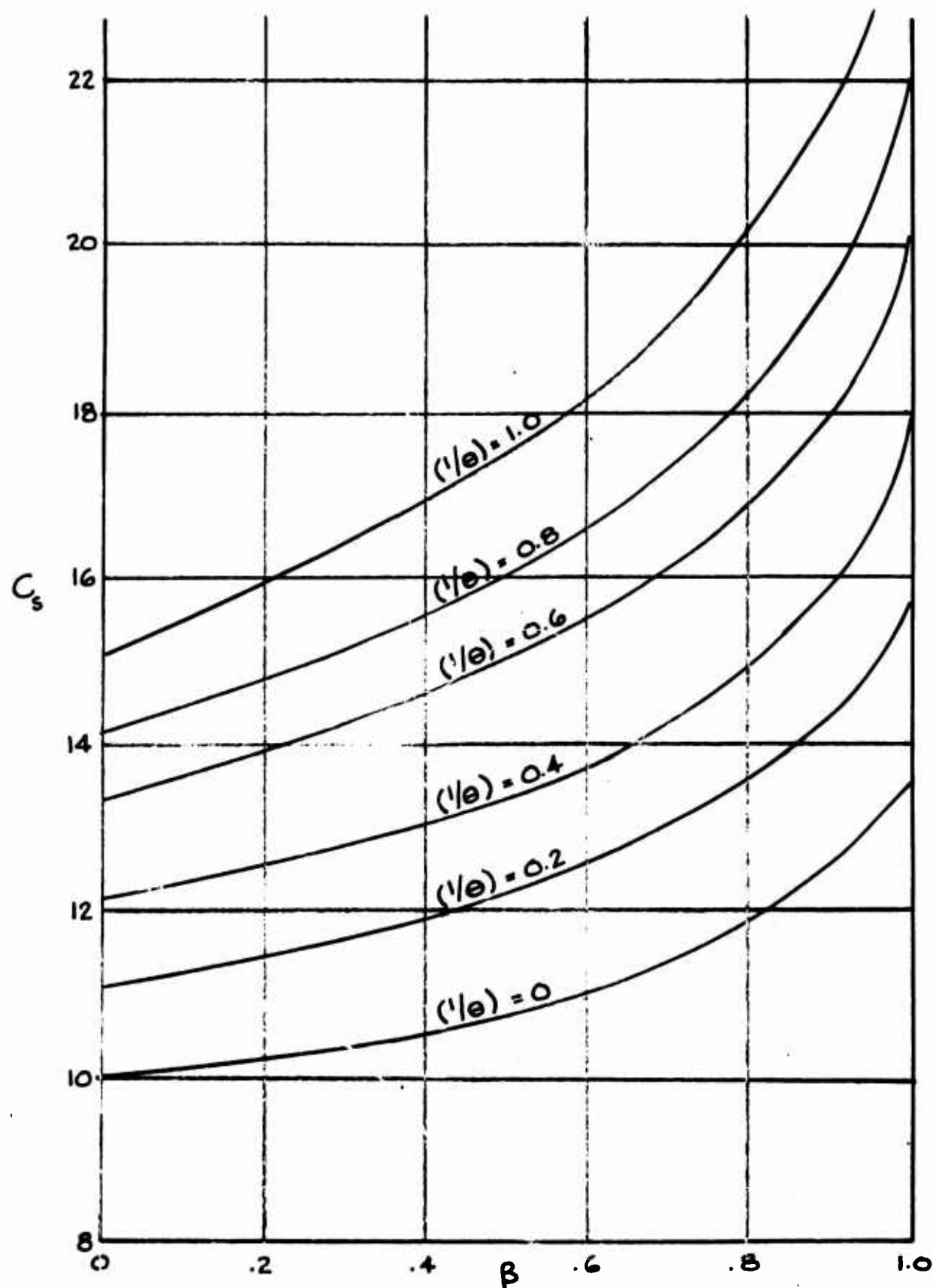


Figure 23
Shear Buckling Parameter

4.4 Two Edges Clamped, Two Edges Simply Supported

The rectangular orthotropic plate with two opposite edges clamped and the other two edges simply supported (Figure 24) is now considered.

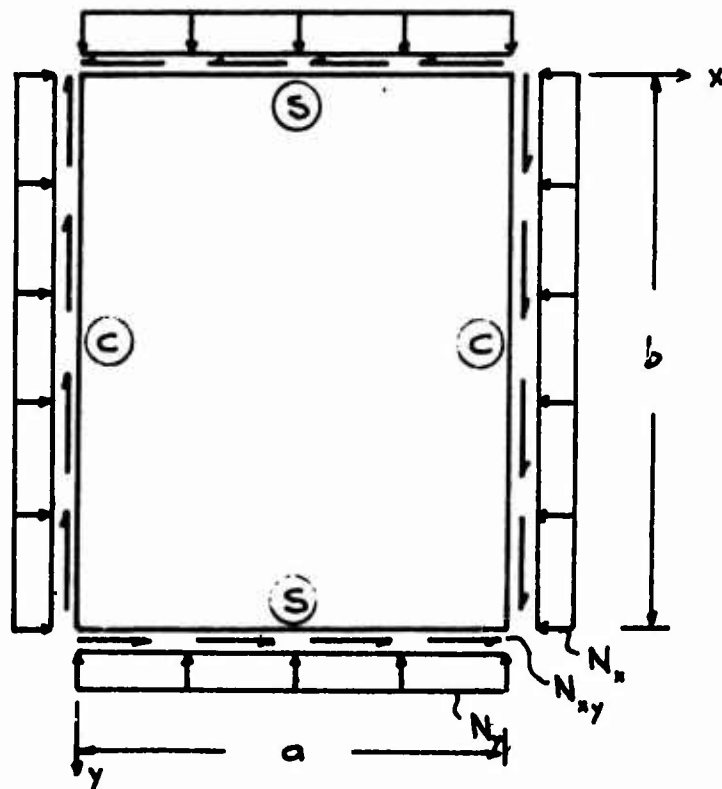


Figure 24
Buckling of © - © - © - © Plate

4.4.1 Clamped Edges Loaded; Others Unloaded

When N_x acts, and $N_y = N_{xy} = 0$, on the plate of Figure 24, Wittrick (Ref. 107) gives the solution as:

$$(N_x)_{cr} = \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \left[k - 2 + \frac{D_x}{\sqrt{D_x D_y}} \right], \quad (146)$$

where k is the coefficient from Figure 21 for a particular value of $(a/b)_e$ given by Eq. (137).

4.4.2 Simply Supported Edges Loaded; Others Unloaded

When N_y acts, and $N_x = N_{xy} = 0$, on the plate of Figure 24, the solution of Wittrick is given in the form:

$$(N_y)_{cr} = \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \left[k - 2.4 + \frac{1.2 D_x}{\sqrt{D_x D_y}} \right], \quad (147)$$

and k is available from Figure 21 for the effective aspect ratio given by Eq. (137).

Heck and Ebner (Ref. 30) give as the solution for the case of $D_x > D_y$,

$$(N_y)_{cr} = \frac{2\pi^2}{b^2} \left[2D_x \frac{b^2}{a^2} + D_y \right]. \quad (148)$$

Lundquist and Stowell (Ref. 253) show that the critical buckling stress for this case is

$$\sigma_{cr} = \frac{\pi^2 t^2}{12 b^3} \left[4.554 \sqrt{D_x D_y} + 1.237 (\mu_{xy} D_x + \mu_{yx} D_y) + 4.943 G \right] , \quad (149)$$

for the minimum condition,

$$\frac{a}{b} = .664 \sqrt{\frac{D_x}{D_y}} .$$

4.4.3 Biaxial Compression

When N_x and N_y act on the plate of Figure 24 and $N_{xy} = 0$, Wittrick uses the curves of Figure 21, but the aspect ratio given by Eq. (140) must be applied. The solution is obtained from Eq. (141).

4.4.4 Edge Shear

As in paragraph 4.3.3, the critical shear force per unit length for the plate of Figure 24, when $N_x = N_y = 0$, is given by

$$(N_{xy})_{cr} = k_s \frac{\pi^2 \sqrt{D_x D_y}}{b^2} ,$$

where the shear buckling coefficient is available from Figure 22b (given by Timoshenko and Woinowsky-Krieger in Reference (97) for plates whose edges obey the inequality,

$$a > b .$$

4.5 All Edges Clamped

A plate whose four edges are clamped (Figure 25) is considered.

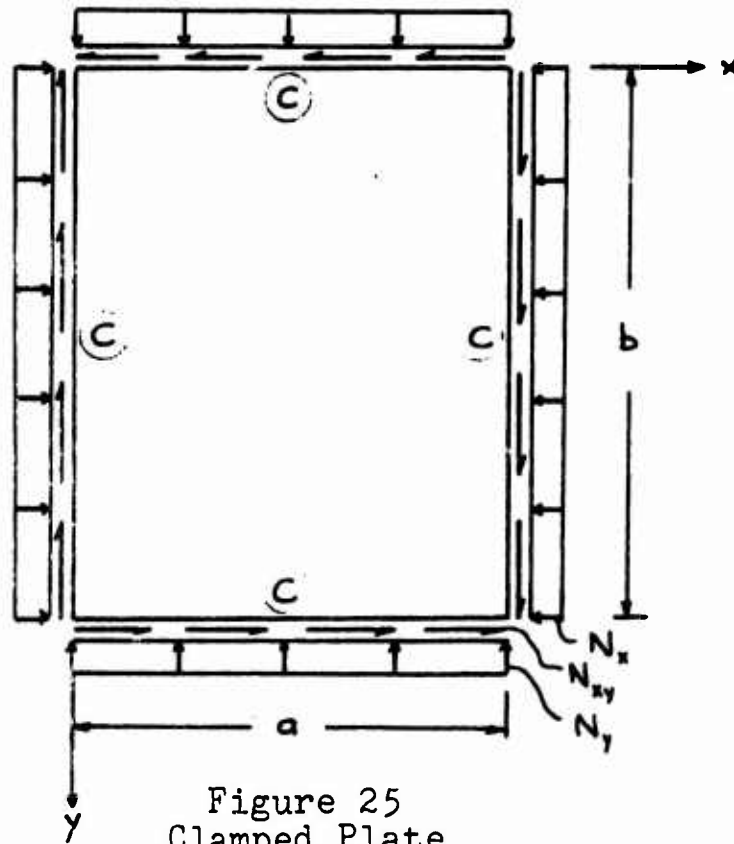


Figure 25
Clamped Plate

4.5.1 Uniaxial Compression

When N_x is acting and $N_y = N_{xy} = 0$, the critical load on the plate of Figure 25 is given by Wittrick (Ref. 107) as

$$(N_{xy})_{cr} = \frac{\pi^2 \sqrt{D_x D_y}}{b^2} \left[k - 2.46 + \frac{1.23 D_x}{\sqrt{D_x D_y}} \right], \quad (150)$$

and the buckling coefficient is given by Figure 21 for the effective aspect ratio,

$$\left(\frac{a}{b}\right)_e = \frac{a}{b} \sqrt{\frac{D_y}{D_x}}.$$

4.5.2 Biaxial Compression

Timoshenko and Gere (Ref. 96) solve the problem of a biaxially compressed isotropic plate with clamped edges, assuming the deformation function as

$$w = \frac{\delta}{4} \left(1 - \cos \frac{2\pi x}{a}\right) \left(1 - \cos \frac{2\pi y}{b}\right), \quad (151)$$

and using strain energy to calculate the critical buckling stress. A similar analysis can be performed for the orthotropic plate.

4.6 Other Buckling Cases

Several other cases of loadings and edge conditions are considered in this section.

4.6.1 Combined Shear and Compression

Sandorff (Ref. 315) investigates the buckling of a plate under the action of a compressive load and shear (Figure 26).

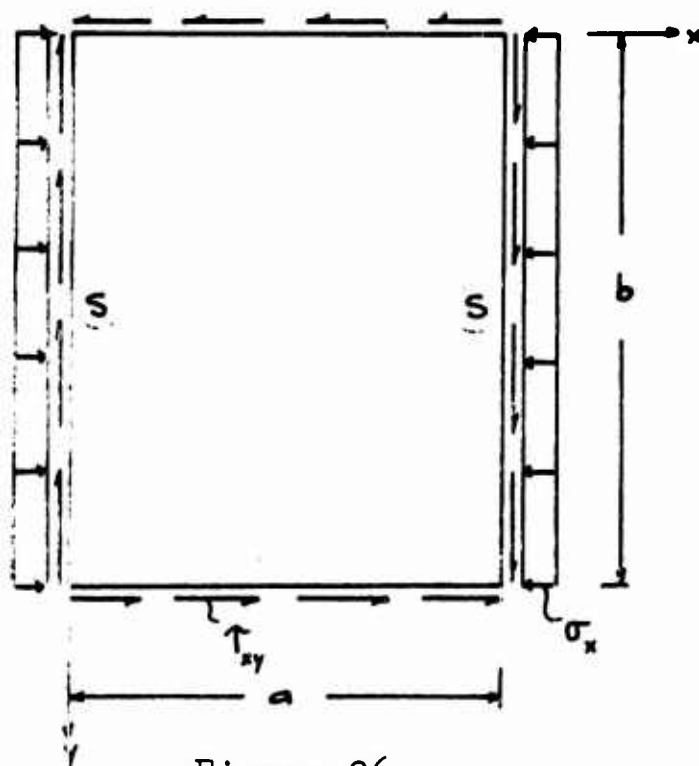


Figure 26
Combined Loading

The assumption is made that b is infinite and an interaction curve (Figure 27) is presented. The parameters of this curve are stress ratios. That is,

$$R_c = \frac{(\sigma_x)_{\text{APPLIED}}}{(\sigma_x)_{\text{ALLOWABLE}}} , \quad R_s = \frac{(\tau_{xy})_{\text{APPLIED}}}{(\tau_{xy})_{\text{ALLOWABLE}}} \quad (152)$$

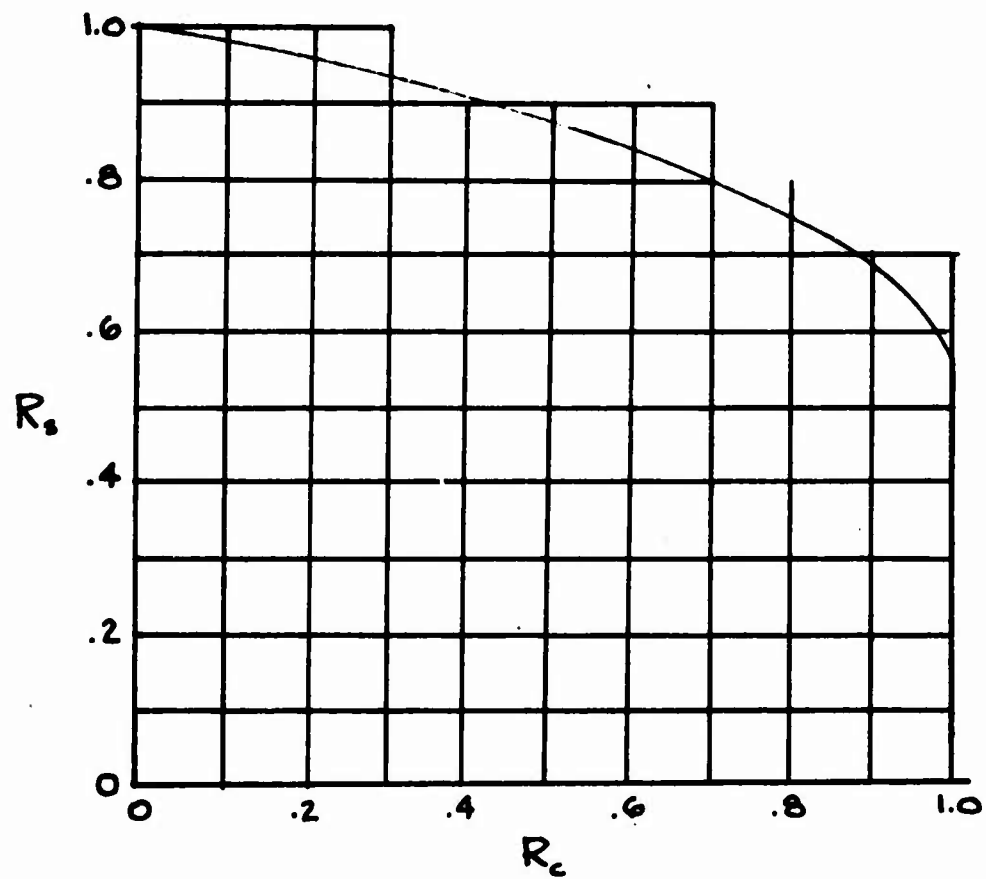


Figure 27
Interaction Shear and Compression

4.6.2 Loaded Edges Simply Supported; Other Edges Clamped and Free

An implicit solution for the case shown in Figure 28 is presented by Lundquist and Stowell (Ref. 253) and discussed by the writer (Ref. 309).

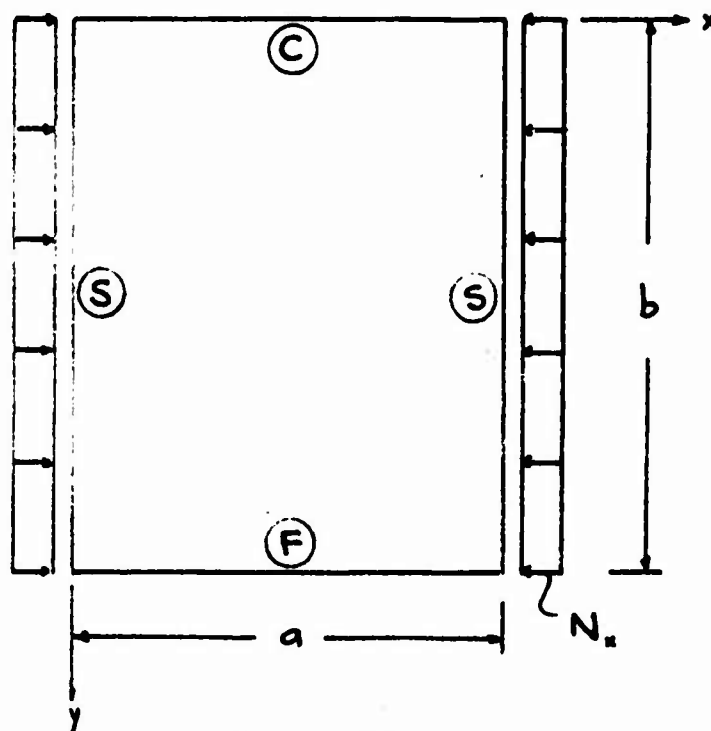


Figure 28
Buckling of (S) - (C) - (S) - (F) Plate

The critical buckling stress is given by:

$$\sigma_{cr} = \left(\frac{t}{b}\right)^2 \left[.769 \sqrt{D_x D_y} - .270 (\mu_{xy} D_x + \mu_{yx} D_y) + 1.712 G \right], \quad (153)$$

for the condition

$$\frac{a}{b} = 1.46 \sqrt{\frac{D_2}{D_1}} .$$

4.6.3 Three Edges Simply Supported; One Edge Free

When the clamped edge of Figure 28 is replaced by a simply supported edge, the critical buckling stress of Eq. (154) simplifies to:

$$\sigma_{cr} = \left(\frac{t}{b}\right)^2 \left[\frac{\pi D_1}{12} \left(\frac{b}{a}\right)^2 + G \right] , \quad (154)$$

for the same limiting condition.

PART 5 - SANDWICH PLATES

The name "sandwich plate" describes a composite plate which consists of two thin sheets--faces-- and a relatively thick core. The faces are generally of aluminum alloy or fiberglass, while the core can be a light solid material such as plastic and wood or a thin metal sheet formed into a cellular configuration ("honeycomb"). The basic principle of sandwich construction is the reliance upon the outer faces to carry end loads and upon the core to transmit shear and keep the faces a constant distance apart.

Ordinarily, the modulus of elasticity of the core material is extremely small (about 0.1%) in comparison to that of the faces. Thus, the core provides very little resistance to bending. But, because its modulus of rigidity is also very low, the core experiences appreciable shearing deformations. Thus, shearing deformations must be considered in the analysis. In addition, since the faces or the core, or both, may have orthogonally anisotropic stretching properties, the composite plate may well be orthogonally anisotropic in its bending properties. Thus, in many cases, the theory

which should be used to analyze sandwich plates is orthotropic plate theory, with consideration of shearing deformations (see Chapter 1).

The amount of research performed in the field of sandwich construction is overwhelming. For this reason, the bibliography of this report, pertaining to this area, is by no means complete. For specialized and more detailed bibliographies of sandwich-plate literature, see References 131, 135, and 137.

Probably the most important of the papers regarding the theory of sandwich plates are those of Reissner (Ref. 169), Hoff (Refs. 139, 140), and Libove and Batdorf (Ref. 155). Other authors who have made significant contributions in the investigation of the bending and the buckling of sandwich plates are: Aleksandrova (Ref. 113), Anderson (Ref. 116), Anderson and Updegraff (Ref. 117), Bijlaard (Ref. 120), Cheng (Ref. 125), Dundrova (Ref. 127), Eringen (Ref. 128), Goodier (Ref. 132), Guest and Solvey (Ref. 136), Hoff and Mautner (Ref. 142), Hopkins and Pearson (Ref. 143), Horvay (Ref. 144), Hubka, Dow and Seide (Ref. 145), Kimel (Ref. 149), Lewis (Ref. 154), Libove and Hubka

(Ref. 156), March and Smith (Ref. 161), Mushtari (Ref. 163), Norris (Ref. 164), Raville (Ref. 168), Seide (Refs. 171, and 172), Seide and Stowell (Ref. 173), Thurston (Ref. 176), Weikel and Kobayashi (Ref. 178), Williams (Ref. 181), Yen, Salerno and Hoff (Ref. 184), Yusuff (Refs. 190, 191), and Zahn and Cheng (Ref. 192).

Burns (Ref. 123) and Burns and Skogh (Refs. 212, 213) present minimum weight analyses of sandwich plates. Kuenzi (Ref. 152) presents design criteria. Keer and Lazan (Ref. 148), Mindlin (Ref. 162), and Yu (Refs. 185-189) consider dynamic behavior. Manufacture of sandwich plates is discussed by Lowy and Jaffee (Ref. 157). References 126 and 170 are of considerable value to the designer as handbooks.

This chapter is intended as a brief summary of some of the highlights of the small- and large-deflection theories and local and gross instability of sandwich plates from several of the papers listed in Section B of the Bibliography.

5.1 Flexure

The analysis of sandwich plates in bending follows essentially the same procedure as that discussed in Part 3. However, as was stated previously, the effect of transverse shear deformations must be incorporated into the analysis.

5.1.1 Small Deflections

Williams (Ref. 181), Hopkins and Pearson (Ref. 143), and March (Refs. 158-161) were among the first investigators to account for shear deformations by assuming that two adjacent vertical strips remain vertical after surface displacement, and that it is the shear between the strips which limits the penetration of the sinusoidal face displacement of the interface of core and facing. Thus, the vertical displacement of the face is taken in the form,

$$v_f = A_0 \sin \frac{m\pi x}{L}, \quad (155)$$

while the displacement of the core in the same direction is given by

$$v_c = v_f e^{-ks}. \quad (156)$$

The constant, k , is given by

$$k = \frac{(G_{xz})_c}{(E_z)_c} \left(\frac{m\pi}{L} \right)^2, \quad (157)$$

where

c = subscript referring to core,

E_z = modulus of elasticity in z -direction,

G_{xz} = shear modulus in xz -plane,

L = panel length,

m = number of half-waves in x -direction,

x = longitudinal coordinate,

z = coordinate through thickness.

More general and rigorous treatments are given by Libove and Batdorf (Ref. 155) and Reissner (Ref. 169). Most of the work which follows chronologically is based on the principles cited in these papers.

The Libove-Batdorf paper (see Paragraph 1.8) develops a theory which applies to orthotropic or isotropic plates with homogeneous or nonhomogeneous cores. The expressions derived therein are Eqs. (45), (46) and (47). Assuming that N_x , N_y , and N_{xy} are

constant throughout the plate, these equations are solved for the deflection and the shears:

$$\begin{aligned} [D] w &= - [M] p, \\ [D] Q_x &= - [N] p, \\ [D] Q_y &= - [P] p. \end{aligned} \quad (158)$$

and the differential operators are defined in the following manner:

$$\begin{aligned} [D] = & \frac{D_x D_y}{2 D_{xy}} \cdot \frac{\partial^4}{\partial x^4} + \left[\frac{D_x D_y}{2 D_{xy}} + \frac{D_1 D_2 - \frac{D_x D_y v_{xy}}{2} - \frac{D_1 D_2 v_{yx}}{2}}{D_{xy}} \right] \cdot \frac{\partial^4}{\partial x^2 \partial y^2} \\ & + \left[\frac{D_x D_y}{2 D_{xy}} + \frac{D_1 D_2 - \frac{D_x D_y v_{xy}}{2} - \frac{D_1 D_2 v_{yx}}{2}}{D_{xy}} \right] \cdot \frac{\partial^4}{\partial x^2 \partial y^4} \\ & + \frac{D_x D_y}{2 D_{xy}} \cdot \frac{\partial^4}{\partial y^4} - D_1 \frac{\partial^4}{\partial x^4} - [2 D_x (1 - v_{xy} v_{yx}) + D_1 v_{xy} + D_2 v_{yx}] \\ & \cdot \frac{\partial^4}{\partial x^2 \partial y^2} - D_2 \frac{\partial^4}{\partial y^4} + \frac{D_x D_y}{2 D_{xy} D_{xy}} [N_x \frac{\partial^4}{\partial x^4} + N_y \frac{\partial^4}{\partial x^2 \partial y^2} + 2 N_{xy} \frac{\partial^4}{\partial x^2 \partial y^2}] \\ & + \left[\frac{D_1 D_2 - \frac{D_x D_y v_{xy}}{2} - \frac{D_1 D_2 v_{yx}}{2}}{D_{xy} D_{xy}} \right] [N_x \frac{\partial^4}{\partial x^2 \partial y^2} + N_y \frac{\partial^4}{\partial x^2 \partial y^4} + 2 N_{xy} \frac{\partial^4}{\partial x^2 \partial y^2}] \\ & + \frac{D_x D_y}{2 D_{xy} D_{xy}} [N_x \frac{\partial^4}{\partial x^2 \partial y^4} + N_y \frac{\partial^4}{\partial y^4} + 2 N_{xy} \frac{\partial^4}{\partial x \partial y^3}] \\ & - \left[\frac{D_x (1 - v_{xy} v_{yx})}{2 D_{xy}} + \frac{D_1}{D_{xy}} \right] [N_x \frac{\partial^4}{\partial x^4} + N_y \frac{\partial^4}{\partial x^2 \partial y^2} + 2 N_{xy} \frac{\partial^4}{\partial x^2 \partial y^2}] \\ & - \left[\frac{D_x (1 - v_{xy} v_{yx})}{2 D_{xy}} + \frac{D_2}{D_{xy}} \right] [N_x \frac{\partial^4}{\partial x^2 \partial y^2} + N_y \frac{\partial^4}{\partial y^4} + 2 N_{xy} \frac{\partial^4}{\partial x \partial y^3}] \\ & + (1 - v_{xy} v_{yx}) [N_x \frac{\partial^4}{\partial x^2} + N_y \frac{\partial^4}{\partial y^2} + 2 N_{xy} \frac{\partial^4}{\partial x \partial y}]. \end{aligned}$$

$$[M] = \frac{D_x D_1}{2 D_{q_x} D_{q_y}} \frac{\partial^4}{\partial x^4} + \left[\frac{D_1 D_3 - \frac{D_x D_1 v_{xy}}{2} - \frac{D_x D_3 v_{yx}}{2}}{D_{q_x} D_{q_y}} \right] \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{D_x D_3}{2 D_{q_x} D_{q_y}} \frac{\partial^4}{\partial y^4} \\ - \left[\frac{D_x (1 - v_{xy} v_{yx})}{2 D_{q_x}} + \frac{D_3}{D_{q_y}} \right] \frac{\partial^2}{\partial y^2} - \left[\frac{D_x (1 - v_{xy} v_{yx})}{2 D_{q_y}} + \frac{D_1}{D_{q_x}} \right] \frac{\partial^2}{\partial x^2} + (1 - v_{xy} v_{yx}),$$

$$[N] = \frac{D_x D_1}{2 D_{q_x}} \frac{\partial^5}{\partial x^5} + \left[\frac{D_1 D_3 - \frac{D_x D_1 v_{xy}}{2} - \frac{D_x D_3 v_{yx}}{2}}{D_{q_y}} \right] \frac{\partial^5}{\partial x^3 \partial y^2} + \frac{D_x D_3}{2 D_{q_y}} \frac{\partial^5}{\partial x \partial y^4} \\ - D_1 \frac{\partial^3}{\partial x^3} - [D_x (1 - v_{xy} v_{yx}) + D_1 v_{xy}] \frac{\partial^3}{\partial x \partial y^2},$$

$$[P] = \frac{D_x D_1}{2 D_{q_x}} \frac{\partial^5}{\partial x^4 \partial y} + \left[\frac{D_1 D_3 - \frac{D_x D_1 v_{xy}}{2} - \frac{D_x D_3 v_{yx}}{2}}{D_{q_x}} \right] \frac{\partial^5}{\partial x^2 \partial y^3} + \frac{D_x D_3}{2 D_{q_x}} \frac{\partial^5}{\partial y^5} \\ - D_3 \frac{\partial^3}{\partial y^3} - [D_x (1 - v_{xy} v_{yx}) + D_3 v_{yx}] \frac{\partial^3}{\partial x^2 \partial y}.$$

The elastic constants which appear in the above equations are defined by Eqs. (20), (21), (22), (38), (39), and (40). A particular problem, thus, is solved by using Eqs. (158) for a given set of boundary conditions. At a free y-edge:

$$M_x = M_{xy} = Q_x = 0. \quad (159)$$

For a simply supported y-edge, with all points* along boundary prevented from moving parallel to edge:

$$w = M_y = \frac{Q_y}{D_{\theta_y}} = 0. \quad (160)$$

For a simply supported y-edge, with all points free to move parallel to edge:

$$w = M_x = M_{xy} = 0. \quad (161)$$

For a clamped y-edge, with movement parallel to edge prevented:

$$w = \left(\frac{\partial w}{\partial x} - \frac{Q_x}{D_{\theta_x}} \right) = \frac{Q_y}{D_{\theta_y}} = 0. \quad (162)$$

For a clamped y-edge, with all points* free to move parallel to edge:

$$w = \left(\frac{\partial w}{\partial x} - \frac{Q_x}{D_{\theta_x}} \right) = M_{xy} = 0. \quad (163)$$

Particular results for the bending of sandwich plates under uniform and concentrated loads are analyzed by Yen, Gunturkun, and Pohle (Ref. 183). However, both the core and the faces are considered isotropic and, therefore, will not be discussed here.

*Except those in middle surface.

The same is true of Kuenzi's (Ref. 152) report on design criteria.

Raville (Ref. 168) presents a study of simply supported rectangular sandwich plates, with isotropic facings and orthotropic core, acted upon by a uniformly distributed load, p . The expression for the maximum deflection (at the center of the plate) is given by a double Fourier series:

$$w_{max} = \frac{16 p a^4 (1-\mu^2)}{\pi^6 EI} \sum_m \sum_n (-1)^{\frac{m+n}{2}-1} \cdot \left\{ \frac{1 + [m^2 + (\frac{1-\mu^2}{2}) n^2 \rho^2] S_x + [(\frac{1-\mu^2}{2}) m^2 + n^2 \rho^2] S_y + [(\frac{1-\mu^2}{2}) (m^2 + n^2 \rho^2)^2] S_x S_y}{mn (m^2 + n^2 \rho^2)^2 [1 + (\frac{1-\mu^2}{2}) (m^2 S_y + n^2 \rho^2 S_x)]} \right\} \quad (164)$$

where

$$S_x = \frac{\pi^2 E_c t_1 t_2}{G_{xx} a^2 (1-\mu^2) (t_1 + t_2)},$$

$$S_y = \frac{\pi^2 E_c t_1 t_2}{G_{yy} a^2 (1-\mu^2) (t_1 + t_2)},$$

$$I = \frac{t_1 t_2}{t_1 + t_2} \left(c + \frac{t_1 + t_2}{2} \right)^2,$$

$m, n = \text{odd integers,}$

$E = \text{Young's modulus of facings,}$

$\mu = \text{Poisson's ratio of facings,}$

$\rho = a/b,$

while a, b and the coordinate axes are consistent with their definitions throughout this report.

In further refinements of small deflection theory, Hoff (Ref. 139) presents a variational approach which is also presented, in a generalized fashion, by Eringen (Ref. 128). Both papers are limited to sandwiches consisting of facings and core with isotropic properties.

Cheng (Ref. 125) considers the problem of bending of a plate with isotropic facings, but orthotropic core. The basic equation for bending is written:

$$\left. \begin{aligned} & \left[1 - D_{yy} \frac{\partial^2}{\partial x^2} - D_{xx} \frac{\partial^2}{\partial y^2} \right] \Delta \Delta w \\ & = \frac{1}{D} \left[\left(1 - D_2 \frac{\partial^2}{\partial x^2} - D_1 \frac{\partial^2}{\partial y^2} \right) P + \frac{2D_{xx}D_{yy}}{1-\mu} \Delta \Delta P \right], \end{aligned} \right\} (165)$$

in which

$$D = \frac{Et (h+t)^2}{2 (1-\mu^2)} ,$$

$$D_{xx} = \frac{(1-\mu) D}{2 h G_{xx}} ,$$

$$D_{yy} = \frac{(1-\mu) D}{2 h G_{yy}} ,$$

$$D_1 = D_{xx} + \frac{2 D_{yy}}{1-\mu} ,$$

$$D_2 = D_{yy} + \frac{2 D_{xx}}{1-\mu} ,$$

h = thickness of core,

t = thickness of facings,

E = Young's modulus of facings,

μ = Poisson's ratio of facings,

$$\Delta = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) .$$

The homogeneous solution of Eq. (165) satisfies

$$\left[1 - D_{yy} \frac{\partial^2}{\partial x^2} - D_{xx} \frac{\partial^2}{\partial y^2} \right] \Delta \Delta w = 0 . \quad (166)$$

This form is used to solve the problem of a simply supported plate under a uniformly distributed load (Raville's solution is given elsewhere in this section). Since the load intensity, p , is constant, the right hand side of Eq. (165) reduces to p/D , and its particular solution is taken in the form:

$$\begin{aligned} w_1 &= \frac{p}{24D} (x^4 - 2ax^3 + a^3x) \\ &= \frac{4pa^4}{\pi^5 D} \sum_m \frac{1}{m^5} \sin \frac{m\pi x}{a} \end{aligned} \quad \left. \vphantom{\sum_m} \right\} (167)$$

The maximum deflection, at the center of the plate, is then given by:

$$w_{\max} = \frac{4pa^4}{\pi^5 D} \sum_m \frac{(-1)^{\frac{m-1}{2}}}{m^5} \left[1 - \frac{2 + \alpha_m \phi_m \tanh \alpha_m}{2 \cosh \alpha_m} \right], \quad (168)$$

where

$$\alpha_m = \frac{m\pi b}{2a},$$

$$\phi_m = \frac{[2\mu D_{xx} - (1+\mu)D_{yy}]\alpha_m^2 + (1-\mu)(b/2)^2}{(D_{yy} - \mu D_{xx})\alpha_m^2 + (1+\mu)(D_{xx} - D_{yy})\alpha_m^2 + (1-\mu)(b/2)^2}.$$

5.1.2 Large Deflections

Generally speaking, the study of large deflections of sandwich plates is an extension, through the inclusion of shear effects, of von Karman's nonlinear theory of elasticity. Such a study is quite important because, since the advent of high strength alloy steels and titanium alloys, it is possible to construct a sandwich consisting of a soft core and extremely thin facings.

Reissner (Ref. 169) considers a plate with isotropic facings and core, and uses the simplifying assumption that stresses in the core, whose vectors lie in the xy-plane (see Fig. 1), and the variation of face stresses over the thickness of the faces, can both be neglected. In other words, the face sheets act as membranes, while the core resists transverse shear and normal stress. Introducing an Airy stress function, F , two simultaneous differential equations are written to describe the behavior of an isotropic sandwich plate:

$$\Delta\Delta F = 2tE \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial y^2} \right) \right], \quad (169)$$

The nonlinear theory of a symmetrically loaded sandwich beam is considered by Zahm and Cheng in Reference 192, based on the assumptions that the exact load distribution is known and can be represented by a Fourier series and that the core material is anti-plane.

5.2 Elastic Stability

Buckling of sandwich plates is divided here into two parts, local instability and general (gross) buckling.

5.2.1 Local Buckling

5.2.1.1 Face Wrinkling

Several investigators consider local crinkling of the faces in a plate consisting of isotropic facings and an orthotropic core. Among those who originated such studies are Williams (Ref. 181), Hoff and Mautner (Refs. 141, 142), and Goodier (Ref. 132). A relatively simple formula for the critical compressive load (applied in x-direction) per unit

width of panel is given by Williams:

$$(N_x)_{cr} = 1.7t \left[1 + \frac{h(E_x)_c}{2tE} \right] \sqrt[3]{G_{xz}(E_x)_c E} \cdot \left[\frac{1}{1 + .29 \left[\frac{G_{xz}}{E} \right]^{3/2} \left[\frac{(E_x)_c}{G_{xz}} \right]^{1/2} \frac{(E_x)_c}{G_{xz}}} \right], \quad (172)$$

where the subscript, c, denotes "core", the coordinates are those of Figure 1, and all other symbols are as defined in Part 5.1.

Goodier and Hsu (Ref. 133) investigate face wrinkling in a non-sinusoidal mode and derive a lower critical load. Yusuff (Refs. 190, 191) considers initial imperfections and, for the plate shown in Figure 29, derives equations for critical stress, based on local wrinkling, of a plate with isotropic faces and core.

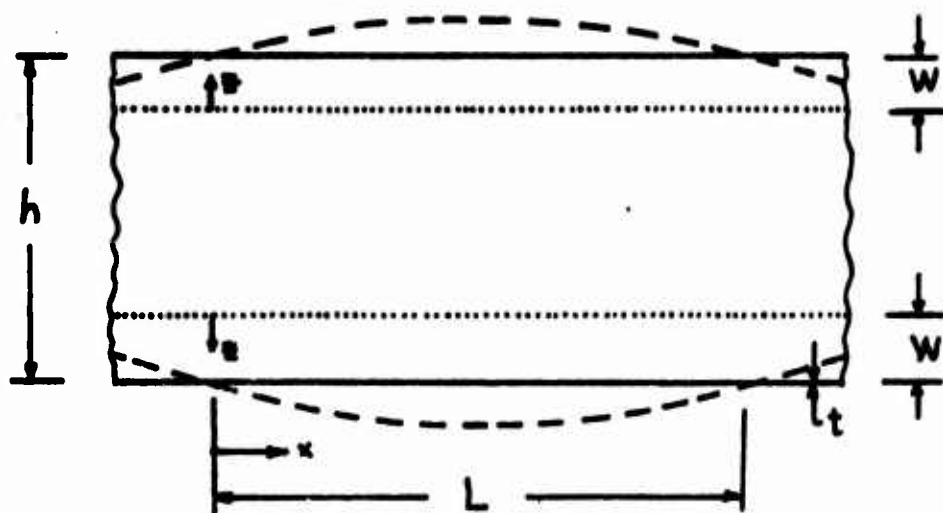


Figure 29
Wrinkling of Facing

W is the "zone of displacement",

$$W = .72t \sqrt{\frac{EE_c}{G_m}}. \quad (173)$$

For a failure of the core by tension or compression,
the critical stress,

$$\sigma_{cr} = \frac{\sqrt{\frac{2EE_c t}{3h}}}{1 + \frac{2E_c}{t} \frac{A_o}{F}} \quad , \quad (t < 2W) \quad (174)$$

$$\sigma_{cr} = \frac{.96 \sqrt[3]{EE_c G_{rs}}}{1 + \frac{E_c}{W} \frac{A_o}{F}} \quad (t > 2W)$$

For core failure due to shear,

$$\sigma_{cr} = \frac{.96 \sqrt[3]{EE_c G_{rs}}}{1 + \frac{\pi G_{rs}}{L_{cr}} \frac{A_o}{F}} \quad (175)$$

The ratio, A_o/\sqrt{F} , is known as the "waviness parameter" and is given in Figure 30. The curve given in this figure has been confirmed by various tests, among them those of Harris and Crisman (Ref. 138).

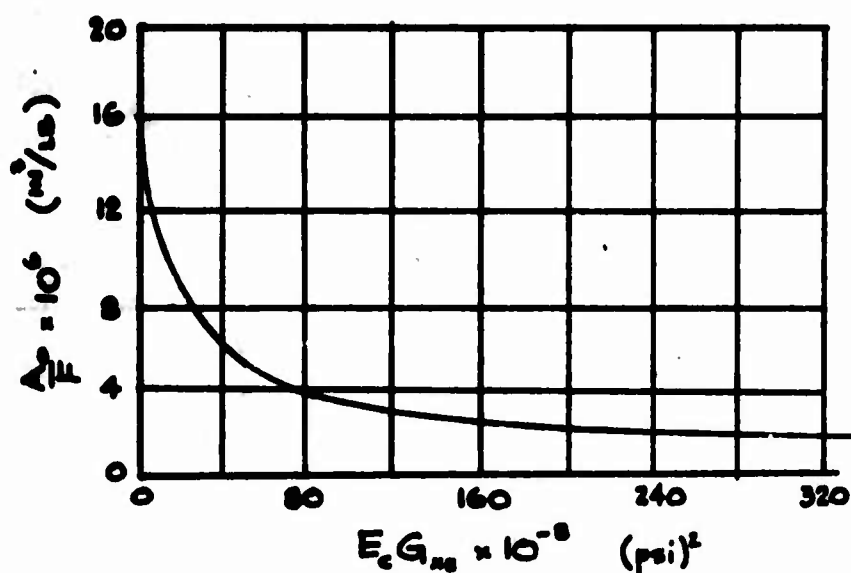


Figure 30
Waviness Parameter versus Core Moduli

5.2.1.2 Intracellular Buckling

A second form of local instability is that of "dimpling" or intracellular buckling. This failure, of course, is dependent upon the core configuration.

Anderson and Updegraff (Ref. 117) investigated the buckling of truss-core sandwich panels experimentally.

$$D_{\Delta\Delta}w = \left[1 - \frac{thE}{2(1-\mu^2)G_c} \Delta \right] \left[P + \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right], \quad (170)$$

where G_c is the shear modulus of the core and the definitions of all other terms are given in Eq. (165).

These equations (127 and 128), together with the boundary conditions, determine the functions F and w . For the solution of such differential equations, the reader should refer to page 425 of Reference 96.

The large deflections of a sandwich plate with orthotropic core between isotropic face sheets are considered by Alwan (Refs. 114, 115). Here, the solution is given by two differential equations, the first of which is Eq. (169) and the second,

$$\left[1 - D_{xx} \frac{\partial^2}{\partial x^2} - D_{yy} \frac{\partial^2}{\partial y^2} \right] \Delta\Delta w = \frac{1}{D} \left[1 - \left(D_{xx} + \frac{2D_{yy}}{1-\mu} \right) \frac{\partial^2}{\partial x^2} - \left(D_{yy} + \frac{2D_{xx}}{1-\mu} \right) \frac{\partial^2}{\partial y^2} + \frac{2D_{xy}D_{yy}}{1-\mu} \Delta\Delta \right] \left[P + \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right]. \quad (171)$$

Writing the critical buckling stress of the faces in the intracellular mode,

$$\sigma_{cr} = \frac{k\pi^2 E_c}{12(1-\mu_c^2)} \left(\frac{t}{b}\right)^2, \quad (176)$$

where the symbols are defined in Figure 31 and in Part 5.1. The values of the buckling coefficient, k , are given in Figure 32.

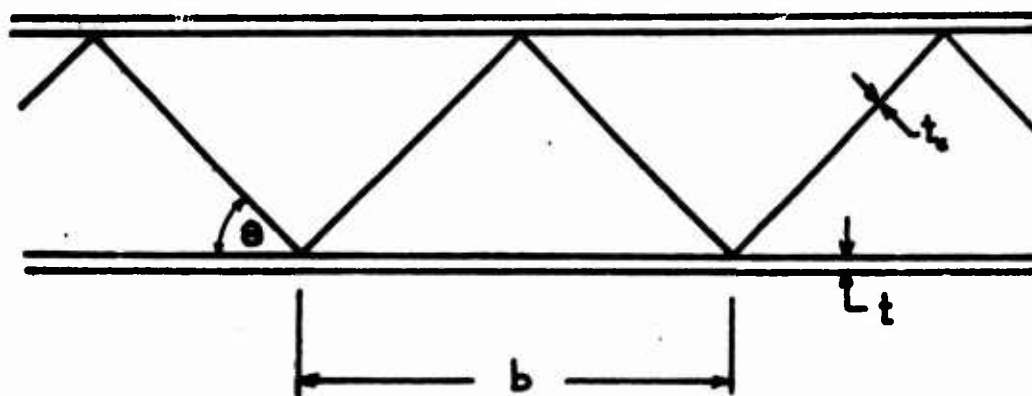


Figure 31
Truss-Core Sandwich

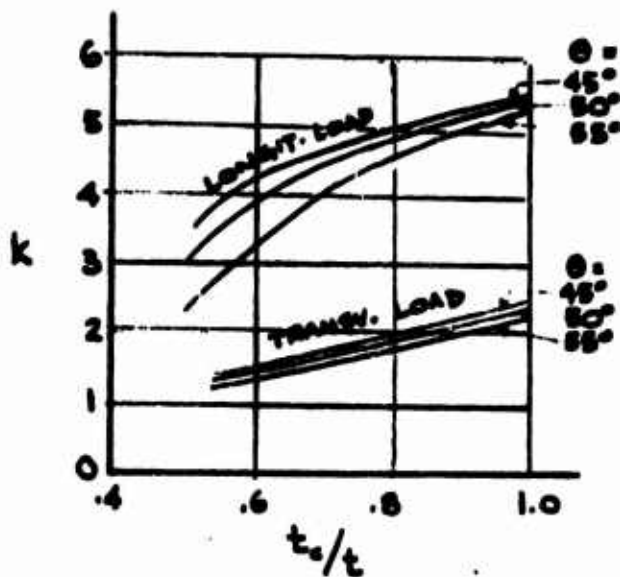


Figure 32

Truss-Core Buckling Coefficient

A design curve, for honeycombs, based on intra-cellular buckling, is given on page C12.8 of Reference 122. When uniaxial compression is applied in the direction of the ribbon,

$$\frac{\sigma_{cr}}{\eta} = .75 E \left(\frac{t}{s} \right)^{\frac{2}{3}}, \quad (177)$$

where η is the plasticity reduction factor,

$$\eta = \frac{2E_T}{E + E_T},$$

and E_T is the tangent modulus of the facings. The geometry of the honeycomb is given in Figure 33.

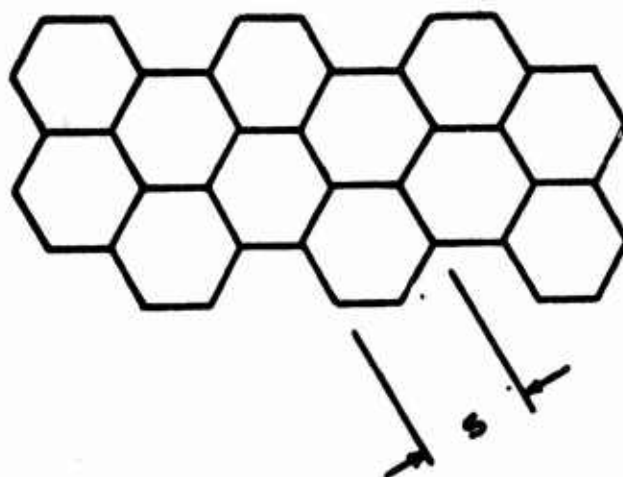


Figure 33
Honeycomb

5.2.1.3 Shear Crimping

The third form of local instability is excessive transverse shear stress (shear crimping). Such a failure may occur when the sandwich panel is not initially perfectly flat.

The problem is considered by Boller and Norris (Ref. 121) and Bruhn (Ref. 122). The latter gives the design equation,

$$\frac{\tau_{cr}}{\eta} = .6E \left(\frac{t}{s} \right)^{3/2}, \quad (178)$$

with terms defined in the same manner as those of Eq. (177).

5.2.2 General Buckling

The general buckling of sandwich plates has been investigated by several authors for many types of end conditions and loadings.

5.2.2.1 Simply Supported Edges

Buckling of simply supported sandwich plates is discussed by Gerard and Becker (Ref. 236), Hoff (Refs. 139, 140), Kimel (Ref. 149), March and Smith (Ref. 161), and Seide and Stowell (Ref. 173).

For the isotropic sandwich plate in uniaxial compression, the formula for the critical buckling stress is given by Hoff:

$$\sigma_{cr} = CF \sigma_{cr}^F \quad (179)$$

σ_{cr}^F is the buckling stress of the unconnected faces,

$$\sigma_{cr}^F = \frac{\pi^2 E t^2}{3(1-\mu^2)L} ,$$

F is the form factor,

$$F = 1 + 3 \left(1 + \frac{h}{t} \right)^2 ,$$

and C is a reduction factor for the buckling of the composite panel. Its values are given by the curve in Figure 34, for various parameters,

$$R = \frac{G_{ss}}{F \sigma_{cr}^F} .$$

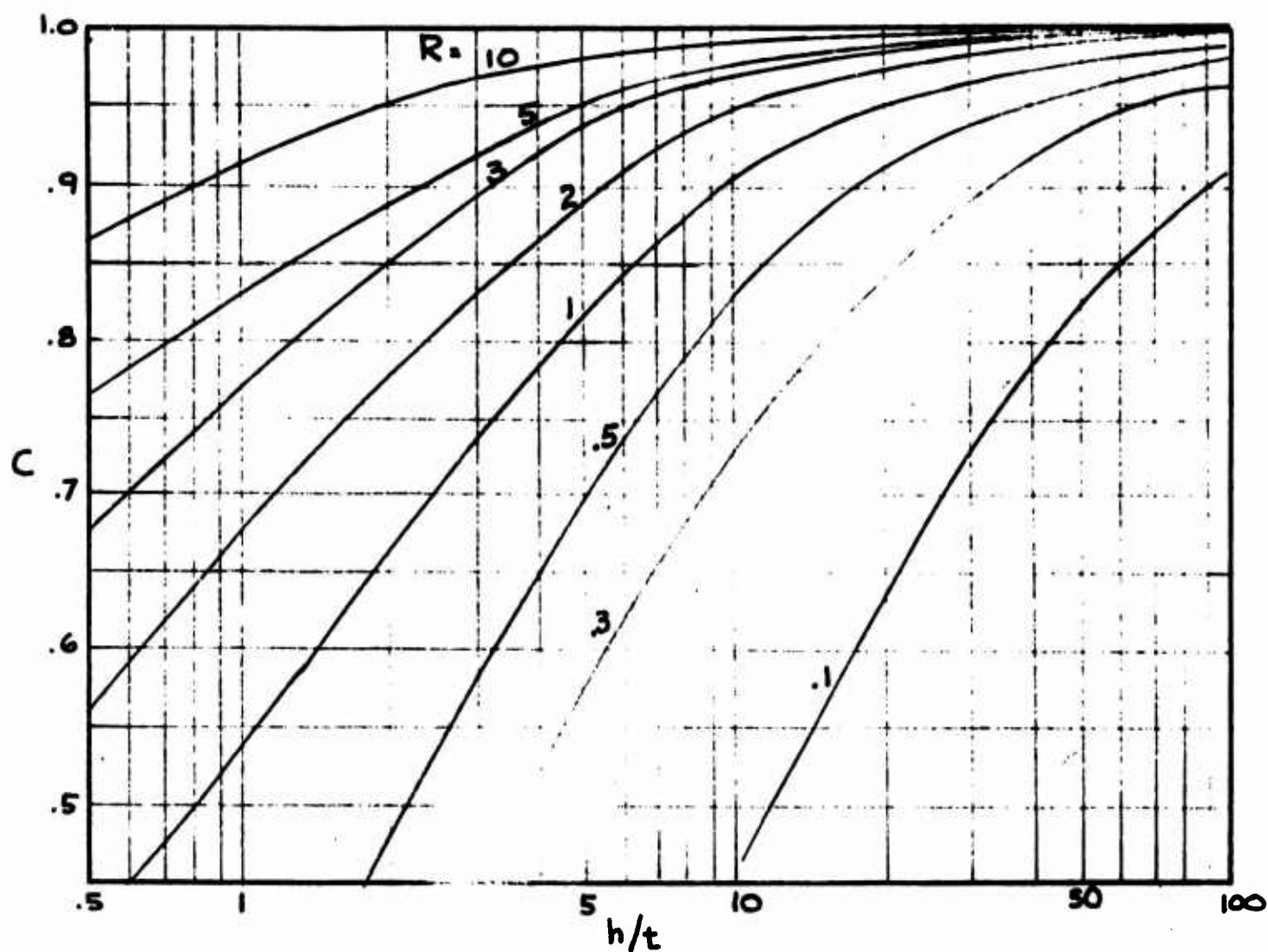


Figure 34

Reduction Factor - Simple Supports

Kimel, in Reference 149, performs a theoretical analysis of the simply supported plate under the combination of edgewise bending and compression on opposite edges. The sandwich considered is made up of isotropic facings and an orthotropic core. The results of this analysis are quite lengthy and will not be presented here.

A uniaxially compressed sandwich with an orthotropic core is discussed by March and Smith (Ref. 161). With the load applied along the a-edges, the critical buckling load,

$$(N_x)_{cr} = \frac{\pi^2}{a^2} \sqrt{D_1 D_2} \left[\frac{b^2}{n^2 a^2} \sqrt{\frac{D_1}{D_2}} + \frac{n^2 a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 2 \Omega \right], \quad (180)$$

where

$$\Omega = (D_1 D_2)^{\frac{1}{2}} \int_{-(\frac{b}{2}+t)}^{(\frac{b}{2}+t)} \left[\frac{E_x \mu_{yx} + 2(1 - \mu_{xy} \mu_{yx}) G_{xy}}{1 - \mu_{xy} \mu_{yx}} \right] z^2 dz,$$

$$D_1 = \int_{-(\frac{b}{2}+t)}^{(\frac{b}{2}+t)} \frac{E_x}{1 - \mu_{xy} \mu_{yx}} z^2 dz,$$

$$D_2 = \int_{-(\frac{b}{2}+t)}^{(\frac{b}{2}+t)} \frac{E_y}{1 - \mu_{xy} \mu_{yx}} z^2 dz,$$

n = number of half-waves in x-direction.

A family of curves, for various values of n , must be constructed in order to make Eq. (180) suitable for design.

5.2.2.2 Loaded Edges Pinned, Other Clamped

This case is considered by Yen, Salerno, and Hoff (Ref. 184). A close approximation to the true value of the buckling load is obtained by establishing a lower bound by Leggett's method and an upper bound by Galerkin's approach. The derived expressions are extremely complex, but a diagram, well suited for design, is presented.

A Metalite sandwich (isotropic facings, orthotropic core) is studied by Seide (Ref. 171). In this paper, the flexural rigidity of the faces is neglected, thus underestimating the shear stiffness of the core (discussed by Bijlaard in Reference 120). Using a combined result of Seide and Bijlaard (modification of former's method by including face rigidities), the critical buckling load along the pinned edges,

$$(N_x)_{cr} = K_o \eta \frac{\pi^2 D_r}{b^2} \left[\frac{D_p}{D_r} + \phi \right], \quad (181)$$

where

K_o = buckling coefficient from Figure 35,

η = plasticity factor (see 5.2.1.2),

$$D_r = \frac{Et(h+t)^2}{2(1-\mu^2)},$$

$$D_p = \frac{Et^3}{6(1-\mu^2)},$$

$$\phi = \frac{\alpha}{\alpha + \eta r K_o},$$

$$r = \frac{(\pi^2 D_r / b^2 G_{xz})}{[h / (h+t)^2]},$$

$$\alpha = 1 + \frac{a^2}{b_m^2},$$

and all other terms are those used with consistency throughout this paper.

March and Smith (Ref. 161) take the x-edges as loaded and simply supported and derive the expression for the critical buckling load of a sandwich with an orthotropic core as:

$$(N_y)_{cr} = \frac{16\pi^2}{3a^4} \left[D_1 \frac{b^2}{n^2 a^2} + \frac{3}{16} D_2 \frac{n^2 a^2}{b^2} + \frac{\Omega}{2} \right], \quad (182)$$

where all terms are defined in Eq. (180).

5.2.2.3 Loaded Edges Clamped, Others Pinned

The solution to the problem of the x-edges loaded and clamped and the y-edges simply supported is given by March and Smith (Ref. 161) for the sandwich with an orthotropic core:

$$(N_y)_{cr} = K_{cr} \frac{12\sqrt{D_1 D_2}}{a^2}, \quad (183)$$

where, for one half-wave,

$$K_{cr} = \frac{\pi^2}{48} \left[3 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 16 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 8 \Omega \right],$$

for two half-waves,

$$K_{cr} = \frac{\pi^2}{60} \left[\frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 41 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 10 \Omega \right],$$

for three half-waves,

$$K_{cr} = \frac{\pi^2}{120} \left[\frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 136 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 20 \Omega \right],$$

for four half-waves,

$$K_{cr} = \frac{\pi^2}{240} \left[\frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 353 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 34 \Omega \right],$$

and all terms are defined in Eq. (180).

5.2.2.4 All Edges Clamped

March and Smith (Ref. 161) solve the problem of the clamped sandwich plate with isotropic facings

and orthotropic core. The critical buckling load along the x-edge,

$$(N_y)_{cr} = K_{cr} \frac{12 \sqrt{D_1 D_2}}{a^2}, \quad (184)$$

in which the buckling coefficient for one half-wave,

$$K_{cr} = \frac{\pi^2}{9} \left[3 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 3 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 2 \Omega \right],$$

for two half-waves,

$$K_{cr} = \frac{\pi^2}{180} \left[16 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 123 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 40 \Omega \right],$$

for three half-waves,

$$K_{cr} = \frac{\pi^2}{45} \left[2 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 51 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 10 \Omega \right],$$

for four half-waves,

$$K_{cr} = \frac{\pi^2}{612} \left[16 \frac{b^2}{a^2} \sqrt{\frac{D_1}{D_2}} + 1059 \frac{a^2}{b^2} \sqrt{\frac{D_2}{D_1}} + 136 \Omega \right],$$

and all terms are defined in Eq. (180).

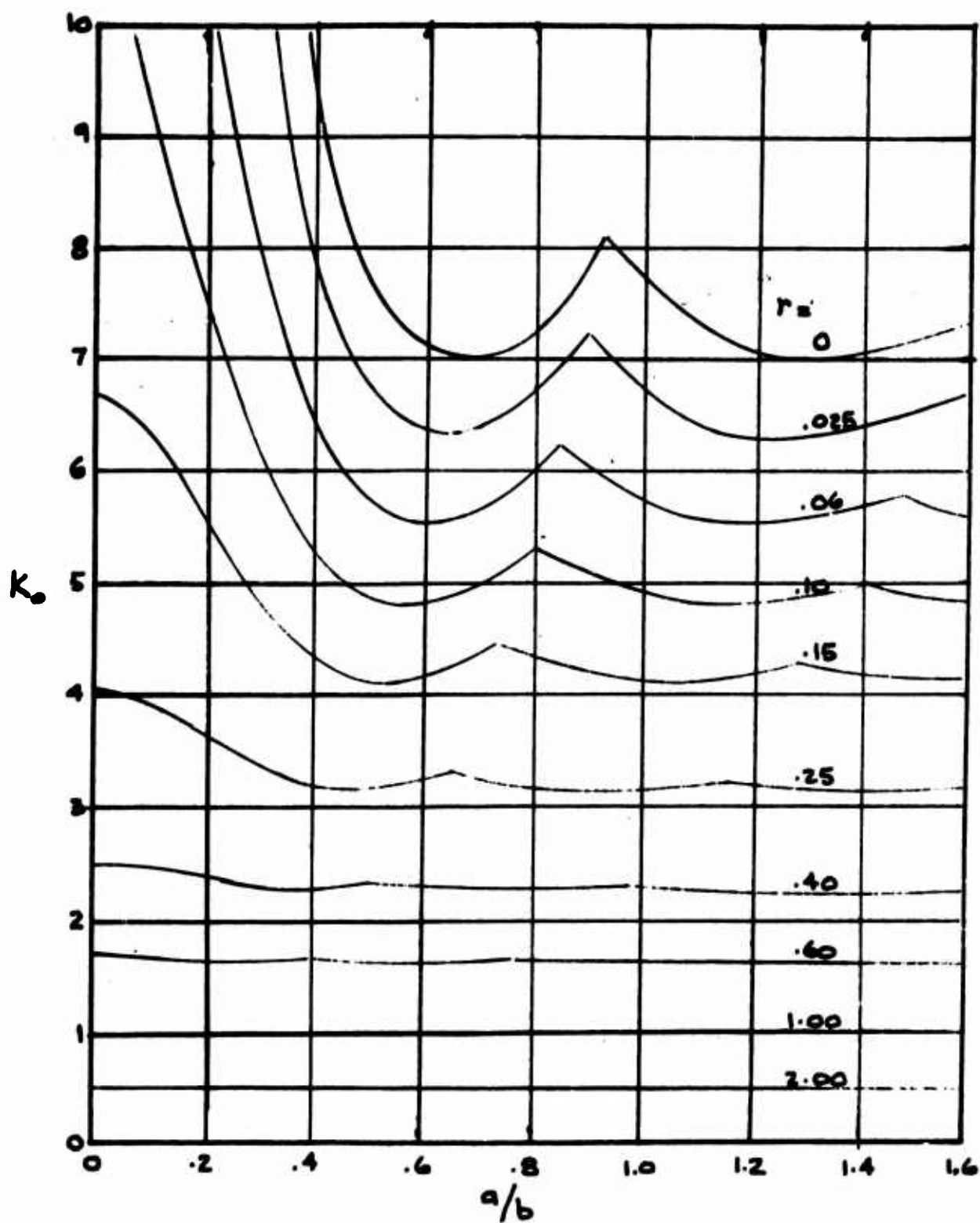


Figure 35

Buckling Coefficients for Eq. (181) $\mu = 1/3$

This problem is also solved by Thurston (Ref. 176). The buckling load is given by:

$$(N_x)_{cr} = K' \frac{\pi^2 E t (h+t)^2}{2 a^2 (1-\mu^2)}, \quad (125)$$

in which K' is a buckling coefficient given by Figure 36. In the Figure, S is the sandwich parameter,

$$S = \frac{D_0 \pi^2}{a^2 G_{xz} h}, \quad (126)$$

where

$$D_0 = \frac{E t (h+t)^2}{2 (1-\mu^2)}.$$

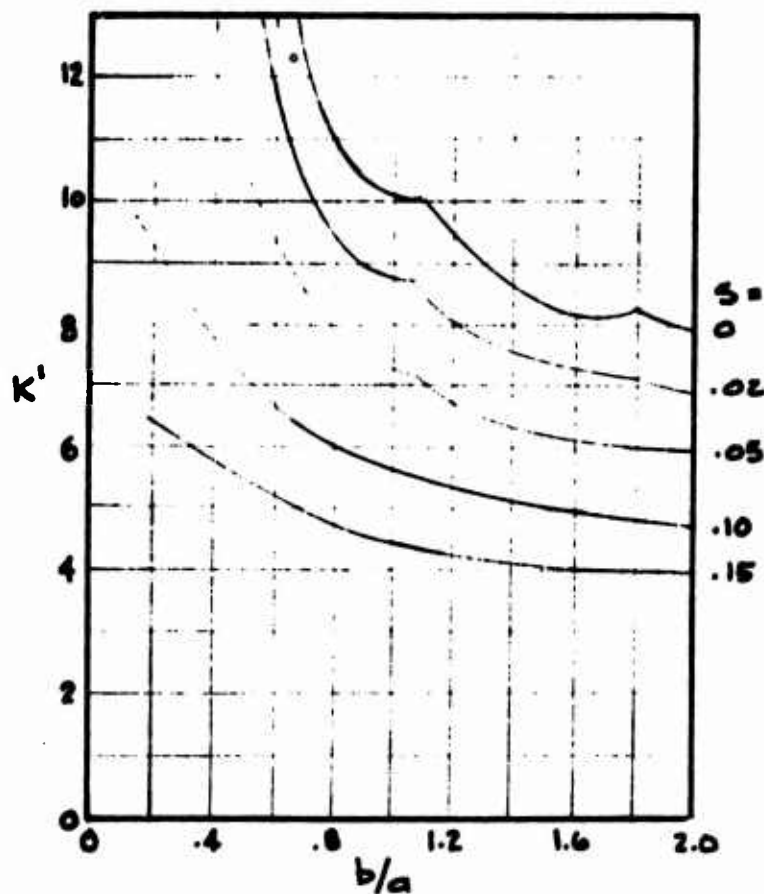


Figure 36

Buckling Coefficients for Eq. (185)

5.3 Summary

Only the most common cases of loading and end supports have been considered in this chapter. For other combinations of end conditions and loads, the reader is advised to search the titles of Section B of the Bibliography.

PART 6 - CORRUGATED PLATES

Corrugated plates were first introduced as a mode of construction in England in the late 1920's. Their first use was in the form of tank and sub-division bulkheads in ships. Corrugated sheet steel was later used in the construction of aircraft wings and fuselages. In recent years, corrugated sheet attached to a flat plate (generally called "corrugation-stiffened panel") has been used in the forming of skin of high-speed flight vehicles such as the X-15. A more recent advance has been the adoption of corrugated plate for use as core in an all-metal sandwich.

6.1 Flexure

The flexure of corrugated plates of various configurations has been discussed by several authors. Among these, the following are the most prominent: Andreeva (Ref. 193), Caldwell (Refs. 196, 197), Shibuya (Ref. 205), Stroud (Ref. 335), Platus and Uchiyama (Ref. 204), Bergman and Reissner (Ref. 195), and Timoshenko and Woinowsky-Krieger (Ref. 97).

The discussion which follows is divided into two parts. First, a single corrugated plate is considered, and, second, a built-up configuration of corrugated

and flat plates is discussed.

6.1.1 Single Sheets

The bending of single corrugated sheet is taken up by Timoshenko and Woinowsky-Krieger (Ref. 97).

For a sinusoidal configuration (Fig. 37), given by

$$z = f \sin \frac{\pi x}{L}, \quad (187)$$

the orthotropic constants are:

$$D_x = \frac{L}{s} \frac{Et^3}{12(1-\mu^2)}, \quad (188)$$

$$D_y = EI, \quad (189)$$

$$D_{xy} = \frac{s}{L} \frac{Et^3}{12(1+\mu)}, \quad (190)$$

in which

E = modulus of elasticity of material,

μ = Poisson's ratio of material,

s = arc length of half-wave.



Figure 37
Sinusoidal Corrugation

From the geometry, the arc length and the moment of inertia are approximately given by

$$s = L \left[1 + \frac{\pi^2 f^2}{4L^2} \right], \quad (191)$$

$$I = \frac{f^3 t}{2} \left[1 - \frac{.81}{1 + 2.5 \left(\frac{f}{2L} \right)^2} \right]. \quad (192)$$

The orthotropic plate theory, discussed in Part 3, is then applied for the solution of a particular bending problem.

Corrugations of a different geometry (Fig. 38) are considered by Caldwell (Ref. 196). The author takes into account the effect of inelastic behavior.

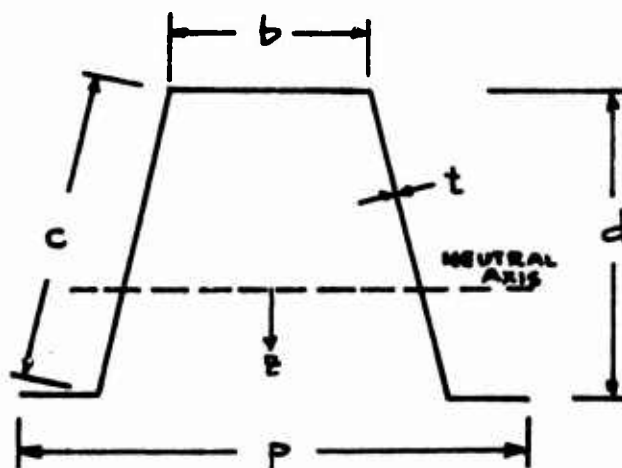


Figure 38
"Hat-type" Corrugation

Experiments show that elastic stresses and deformations can be calculated from ordinary bending theory using a moment of inertia,

$$I = \frac{td^2}{6} (3b + c) , \quad (193)$$

and a section modulus,

$$Z = \frac{td}{3} (3b + c) . \quad (194)$$

Assuming that the strains are large, the ultimate bending strength of the panel is given by:

$$M_{ult.} = \frac{td}{2} (2b + c) \sigma_y , \quad (195)$$

where σ_y is the yield strength of the material.

6.1.2 Built-up Panels

Stroud (Ref. 335) calculates the elastic constants of a corrugated plate attached on one side to a flat plate (Fig. 39).

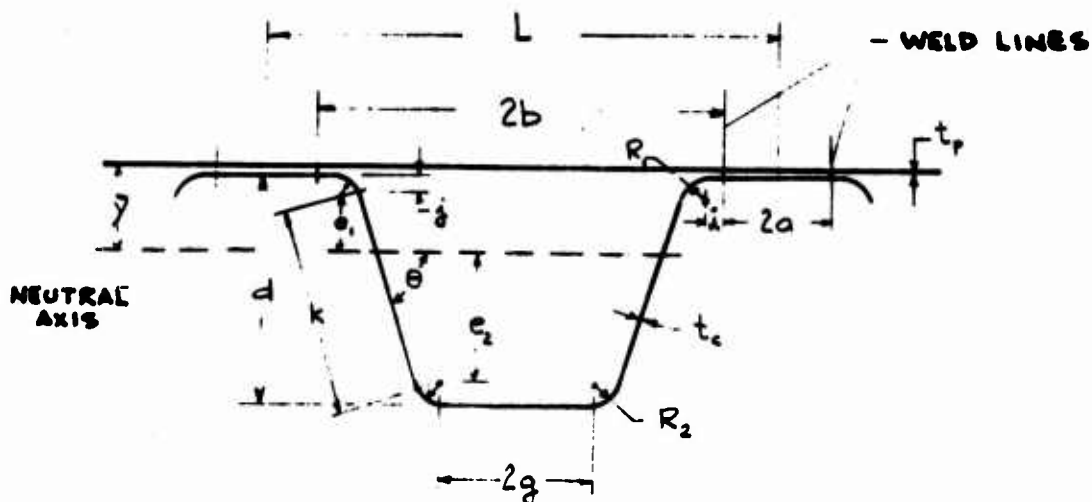


Figure 39
Corrugation-Stiffened Plate

The differential equation of this orthotropic plate is written in a form slightly different from that of Eq. (31):

$$\frac{D_1}{1-\mu_{xy}\mu_{yx}} \frac{\partial^4 w}{\partial x^4} + 2 \left[D_3 + \frac{\mu_{xy} D_1}{1-\mu_{xy}\mu_{yx}} \right] \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_2}{1-\mu_{xy}\mu_{yx}} \frac{\partial^4 w}{\partial y^4} = p. \quad (196)$$

D_1 is the bending stiffness in a direction parallel to the troughs ("into the paper" - - - Fig. 39),

and, with E being the modulus of elasticity of the material, its value is:

$$D_1 = \frac{2E}{L} \left[\frac{(a+b)t_r^3}{12} + (\bar{y} - \frac{t_r}{2})^2(a+b)t_r + \frac{(a+i)t_c^3}{12} \right. \\ + (e_1 + R_1)^2(a+i)t_c + t_c R_1^3 \left(\frac{\Theta}{2} + \frac{\sin 2\Theta}{4} \right) + 2t_c R_1^2 e_1 \sin \Theta \\ + e_1^2 R_1 \Theta t_c + \frac{t_c^3}{12} k^3 \sin^2 \Theta + \frac{k t_c^3}{12} \cos^2 \Theta \\ + \left(t_r + \frac{t_c}{2} + j + \frac{k}{2} \sin \Theta - \bar{y} \right) k t_c + t_c R_2^3 \left(\frac{\Theta}{2} + \frac{\sin 2\Theta}{4} \right) \\ \left. + 2t_c e_c R_2^2 \sin \Theta + e_c^2 R_2 \Theta t_c + \frac{g t_c^3}{12} + (e_c + R_c)^2 t_c g \right], \quad (197)$$

in which the location of the neutral axis is given by

$$\bar{y} = \frac{\left[\frac{t_r^3}{2} (a+b) + (t_r + \frac{t_c}{2})(a+i)t_c + \left(t_r + \frac{t_c}{2} + R_1 - \frac{R_1 \sin \Theta}{\Theta} \right) R_1 \Theta t_c \right. \\ + \left(t_r + \frac{t_c}{2} + j + \frac{n-j}{2} \right) k t_c + \left(t_r + \frac{t_c}{2} + d - R_c + \frac{R_c \sin \Theta}{\Theta} \right) R_c \Theta t_c \\ \left. + \left(t_r + \frac{t_c}{2} + d \right) g t_c \right]}{t_r(a+b) + (a+i)t_c + R_1 \Theta t_c + k t_c + R_c \Theta t_c + g t_c}.$$

The bending stiffness in a direction normal to the trough axis,

$$D_2 = \frac{a+b}{\frac{a}{D_1} + \frac{b}{D_2}}, \quad (198)$$

where

$$D_1' = \left\{ \left[\frac{bEt_p^3}{12(1-\mu^2)} \frac{1}{i + (R_1 + R_2)\theta + k + g - \dots} \right. \right. \\ \left. \frac{-H[R_1^3(\theta - \sin\theta) + jk + \frac{k^3}{2}\sin\theta + \dots]}{+ R_2(n\theta - R_2\theta\cos\theta + R_2\sin\theta) + gd} \right. \\ \left. + \frac{Et_p^3}{12(1-\mu^2)} \right] (1 - \mu_{xy}\mu_{yx}) \right\},$$

$$D_2'' = \frac{E(t_p + t_c)^3}{12(1-\mu^2)} (1 - \mu_{xy}\mu_{yx}),$$

$$H = \frac{R_1^3(\theta - \sin\theta) + jk + \frac{k^3}{2}\sin\theta + \dots}{R_1^3(\frac{3\theta}{2} - 2\sin\theta + \sin\frac{3\theta}{2}) + j^3k + \dots} \\ \frac{+ R_2(n\theta - R_2\theta\cos\theta + R_2\sin\theta) + gd \dots}{+ jk^2\sin\theta + \frac{k^3}{3}\sin^3\theta + R_2n^2\theta - \dots} \\ \frac{- 2nR_2^2\theta\cos\theta + 2nR_2^2\sin\theta + \dots}{+ \frac{R_2^3}{2}(3\theta - 3\sin\theta\cos\theta - 2\theta\sin^2\theta) + gd^2}.$$

The twisting stiffness, with G being the shear modulus of the material,

$$D_2 = \frac{\frac{1}{2} \{ b(t_1 + t_2) + d [b + g - i + (R_2 - R_1) \tan \frac{\theta}{2} + 2(R_2^2 - R_1^2) \tan \frac{\theta}{2} + (R_2^2 - R_1^2) \theta] \}}{\frac{b}{t_1} + \frac{1}{t_2} [i + \theta (R_1 + R_2) + k + g]} \quad (199)$$

The bending moments are evaluated by substituting the above values (in which μ is the Poisson's ratio of the material) into

$$\left. \begin{aligned} M_x &= -\frac{D_1}{1 - \mu_{xy}\mu_{yx}} \left[\frac{\partial^2 w}{\partial x^2} + \mu_{xy} \frac{\partial^2 w}{\partial y^2} \right], \\ M_y &= -\frac{D_1}{1 - \mu_{xy}\mu_{yx}} \left[\frac{\partial^2 w}{\partial y^2} + \mu_{yx} \frac{\partial^2 w}{\partial x^2} \right], \\ M_{xy} &= D_2 \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \right\} \quad (200)$$

The same problem is considered by Wempner and McKinley (Ref. 180) and similar results are derived for plates connected by one weld-line ($a = 0$ in Fig. 39).

6.2 Elastic Stability

The buckling of corrugated plates has been studied by various investigators. Among these are Andreeva (Ref. 193), Ashwell (Ref. 194), Libove and Hubka (Ref. 156), Plaineveaux (Ref. 203), Stroud (Ref. 326), Seydel (Ref. 84), Timoshenko and Gere (Ref. 96), Wempner

and McKinley (Ref. 180), and Dean (Ref. 198).

This portion will again be divided into two phases: the first dealing with single corrugated sheets and the second with panels consisting of flat plates attached to corrugated sheets.

6.2.1 Single Sheets

Since consideration of bending requirements alone would dictate the design of a thin plate with deep troughs, it is necessary to consider limitations imposed upon such a design by local instability of the component "walls". Caldwell (Ref. 196) discusses this problem, taking into account the interaction between webs and flanges and referring all symbols to the configuration of Figure 38.

The critical buckling stress, based on local instability (and, thus, independent of panel length and edge supports), is given by:

$$\sigma_{cr} = K_c^2 \frac{D}{t^3} , \quad (201)$$

where D is a material property, $Et^3/12(1-\mu^2)$, and K_c is a constant dependent upon the dimensions of the corrugation (see Figure 38 and 40).

Timoshenko and Gere (Ref. 96) discuss the work of Bergmann and Reissner (Ref. 195) and Seydel (Ref. 84) for the critical buckling shear load, and give the expression:

$$(N_{xy})_{cr} = 4C_s \frac{\sqrt{D_1 D_2}}{b^2}, \quad (202)$$

where C_s is the buckling coefficient given in Figure 23. The equation (202) is that given in paragraph 4.3.3 and all definitions are given there.

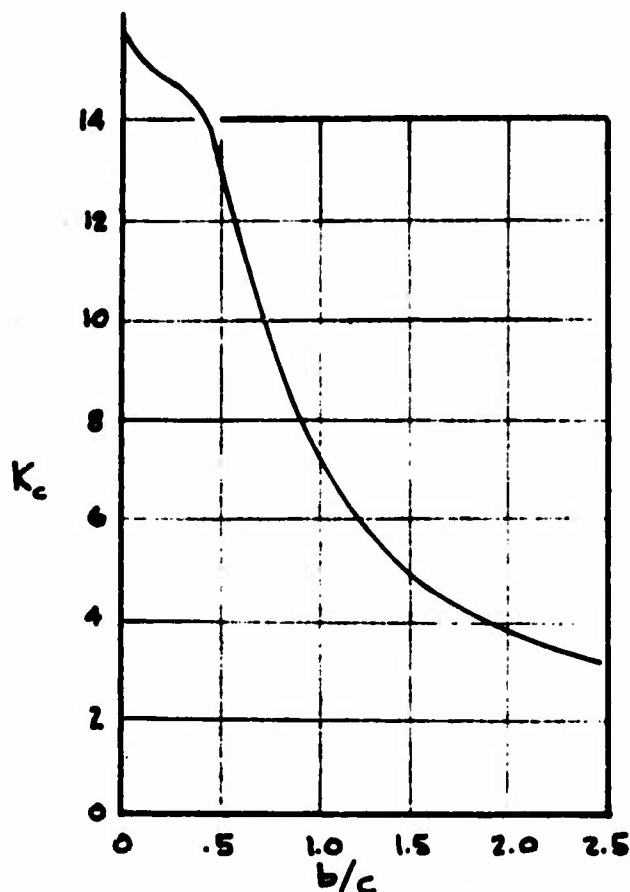


Figure 40
Buckling Coefficient of Eq. (201)

6.2.2 Built-up Panels

The formulas derived by Stroud (Ref. 335) can be extended to the buckling case by rewriting the differential equation (196) such that it includes edge loadings:

$$\begin{aligned} \frac{D_1}{1-\mu_{xy}\mu_{yx}} \frac{\partial^4 w}{\partial x^4} + 2 \left[D_1 + \frac{\mu_{xy} D_1}{1-\mu_{xy}\mu_{yx}} \right] \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_2}{1-\mu_{xy}\mu_{yx}} \frac{\partial^4 w}{\partial y^4} \\ = N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \end{aligned}$$

Thus, if the orthotropic constants defined by Eqs. (197), (198), and (199) are substituted into Eq. (203), the solutions given in Part 4 can be applied to particular problems.

This problem is also discussed by Wempner and McKinley (Ref. 180) and theoretical results are compared with experimental data.

6.3 Summary

The increase in load-carrying ability accomplished by constructing corrugations parallel to the loading and the high flexural rigidity of corrugated plates are well recognized. However, plates of this type have several disadvantages: a) i.e. strength is increased

in one direction, while it is decreased in the direction normal to it; b) weight per unit projected area is greater than that for flat plates; c) workability of corrugated sheet is limited.

For these reasons, various investigators have explored other configurations which can be formed as easily as corrugations. Smith and Gray (Ref. 320) discuss a method for redistributing metal in flat sheets for the purpose of obtaining maximum strength-weight ratios and rigidity-weight ratios. Farmer and Spangler (Ref. 308) investigate a pattern of "calottes", or hemispheres, pressed into a flat sheet in a random pattern. This writer (Ref. 309) and Tewes (Ref. 321) present a study of plates stiffened by "dimples" (L-shaped depressions pressed into the sheet in an interlocking manner). "Waffle beading", forming of intersecting sinusoidally shaped beads in a sheet metal panel, is discussed by Rogge in Reference 314.

PART 7 - RIB-REINFORCED PLATES

The bending and buckling of flat plates stiffened by longitudinal or transverse ribs, or both, is a subject which has received much attention in the literature. Early studies were concerned with multi-piece skin-and-stiffener combinations, while more recent investigations are generally confined to integrally-stiffened structures. Both types are considered in this chapter.

7.1 Flexure

Among the many investigators concerned with the bending of rib-stiffened plates are the following: Borcz (Ref. 326), Chapman and Slatford (Ref. 217), Floor and Burgerhout (Ref. 233), Giencke (Refs. 237-239), Giuliani (Ref. 240), Holmes (Ref. 244), Huber (Ref. 40), Jaeger and Hendry (Ref. 246), Lekhnitsky (Ref. 58), Richart, Newmark and Siess (Ref. 268), Schade (Refs. 273-276), Schumann (Refs. 278, 279), Terazawa and Tarada (Ref. 288), Timoshenko and Woinowsky-Krieger (Ref. 97), Tolotti and Grioli (Ref. 290), and Yamaki (Ref. 298).

The bending of orthogonally stiffened plates (to which this study is limited) can be analyzed by means of orthotropic plate theory, as described in Parts 1 and 2. The elastic constants of the governing differential equation (31) must be determined experimentally, according to the methods outlined in the following chapter. Then, deformations, stresses, etc., can be evaluated. Orthotropic constants for several cases of stiffening are discussed here.

Lekhnitsky (Ref. 58) considers a plate with equally spaced one-directional stiffeners symmetrical with respect to the midplane of the plate (Figure 41).

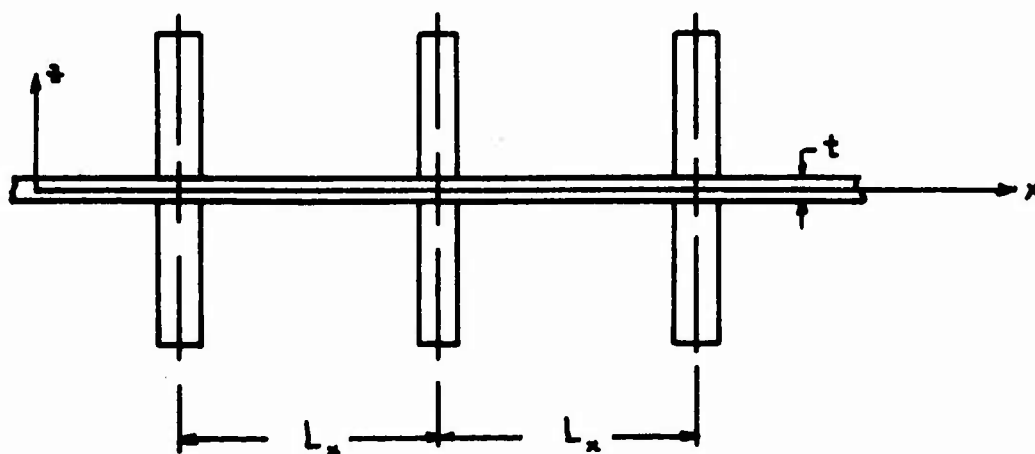


Figure 41
Stiffened Plate Governed by Eqs. (204)

The constants of the characteristic equation (31) are:

$$\left. \begin{aligned} D_x &= D_{xy} = \frac{E_p t^3}{12(1-\mu^2)} , \\ D_y &= \frac{E_p t^3}{12(1-\mu^2)} + \frac{E_s (I_y)_s}{L_x} , \end{aligned} \right\} (205)$$

where subscripts p and s refer to properties of plate and stiffener, respectively.

If the plate of Figure 41 is now reinforced by another set of equal stiffeners in the y-direction, a distance L_y apart,

$$\left. \begin{aligned} D_x &= \frac{E_p t^3}{12(1-\mu^2)} + \frac{E_s (I_x)_s}{L_y} , \\ D_y &= \frac{E_p t^3}{12(1-\mu^2)} + \frac{E_s (I_y)_s}{L_x} , \\ D_{xy} &= \frac{E_p t^3}{12(1-\mu^2)} , \end{aligned} \right\} (206)$$

where

E_p, E_s = moduli of elasticity of plate and stiffeners, respectively,

μ = Poisson's ratio of plate material,

$(I_x)_s$ = moment of inertia of one stiffener, parallel to x-axis,

$(I_y)_s$ = moment of inertia of one stiffener, parallel to y-axis.

The constants for a plate integrally reinforced by equal and equidistant ribs are derived by Pflügger (Ref. 264) and Timoshenko and Woinowsky-Krieger (Ref. 97). The latter give the following expressions for the plate shown in Figure 42:

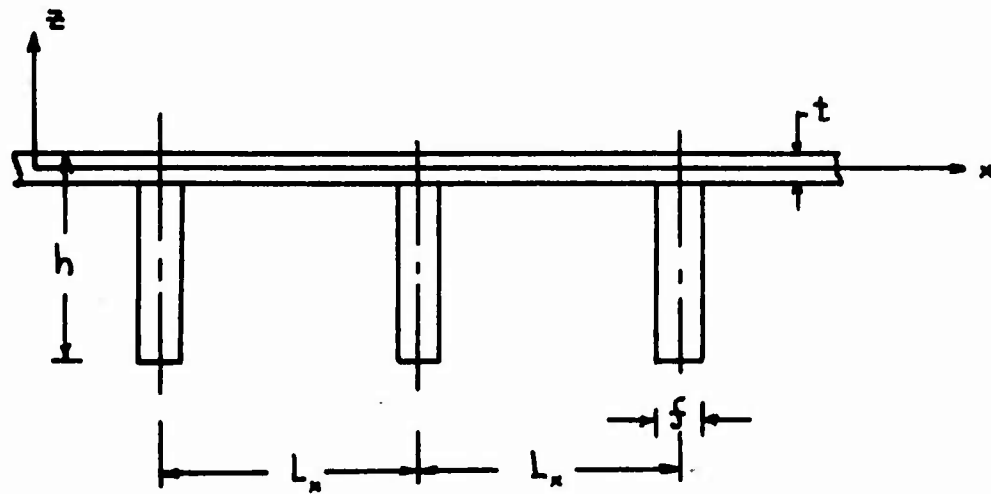


Figure 42
Integrally Stiffened Plates Governed by Eqs. (206)

$$\begin{aligned}
 D_x &= \frac{EL_x t^3}{12(L_x - f + \alpha^2 f)} , \\
 D_y &= \frac{EI_x}{L_x} , \\
 D_{xy} &= 2D'_x + \frac{C}{2L_x} ,
 \end{aligned}
 \quad \left. \vphantom{\begin{aligned} D_x \\ D_y \\ D_{xy} \end{aligned}} \right\} (206)$$

where

I_x = moment of inertia of one T-section
of width L_x ,

C = torsional rigidity of one rib,

D'_x = torsional rigidity of plate without
ribs,

$\alpha = t/d$.

In addition to solutions which make use of orthotropic theory, in which the interaction between plate and stiffener is generally neglected, several writers presented more sophisticated direct methods of attack.

One of the early investigators, Schade (Ref. 276), considers a rectangular plate with longitudinal and transverse stiffeners, and with the two y-edges clamped and the x-edges free, acted upon by a uniformly distributed load, p (Figure 43).

The deformation expression, taken as a single series, has the form:

$$w = \frac{4 p b^4 s_y}{\pi^6 E I_y} \sum_n \frac{1}{n^5} \sin \frac{n \pi y}{b} \left\{ 1 - \frac{1}{\sinh \alpha a + \frac{\alpha}{\beta} \sin \beta a} \right. \\ \cdot [\sinh \alpha (a-x) \cos \beta x + \frac{\alpha}{\beta} \cosh \alpha (a-x) \sin \beta a \\ \left. + \sinh \alpha x \cos \beta (a-x) + \frac{\alpha}{\beta} \cosh \alpha x \sin \beta (a-x)] \right\}, \quad (207)$$

where

$$\alpha = \frac{n}{b} \frac{\pi}{\sqrt{2}} \sqrt{\frac{\dot{\lambda}_y}{\dot{\lambda}_x}} \sqrt{1+\eta},$$

$$\beta = \frac{\sqrt{1-\eta}}{\sqrt{1+\eta}} \alpha,$$

$$\eta = \frac{\dot{\lambda}_x}{\sqrt{\dot{\lambda}_x \dot{\lambda}_y}},$$

$$\dot{\lambda}_x = \frac{I_x}{s_x} + \frac{2(I_m - I_x)}{b} = \text{equivalent moment of inertia of longitudinal stiffener,}$$

$$\dot{\lambda}_y = \frac{\dot{\lambda}_{yx} + \dot{\lambda}_{yy}}{2} = \text{average unit moment of inertia of plate alone,}$$

$$I_x = \text{moment of inertia of longitudinal stiffener and effective skin,}$$

$$I_y = \text{moment of inertia of transverse stiffener and effective skin,}$$

$$I_m = \text{moment of inertia of central longitudinal stiffener and effective skin,}$$

$$\dot{\lambda}_y = \frac{I_y}{s_y} = \text{unit moment of inertia of plate alone.}$$

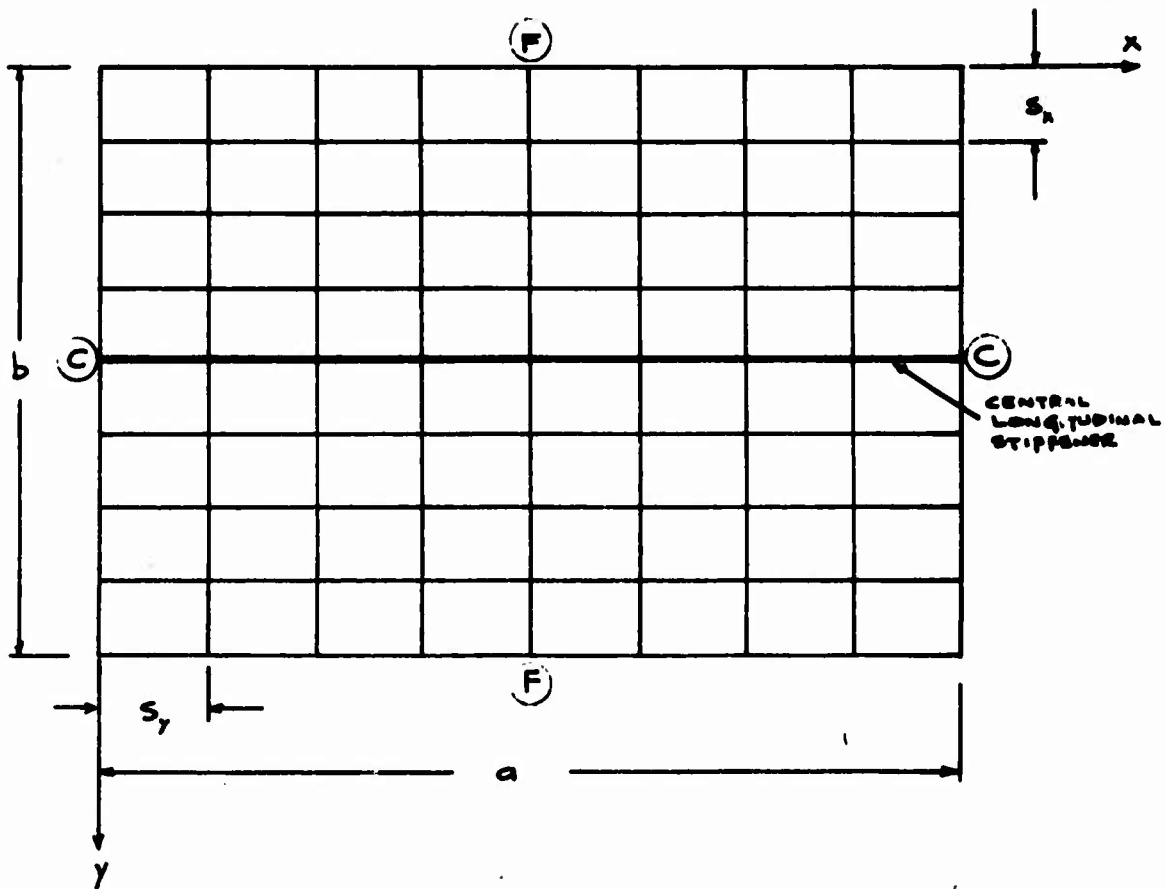


Figure 43
Stiffened Plate Governed by Eq. (207)

A numerical procedure for the direct calculation of stresses is presented by Kempner (Ref. 248). Holmes (Ref. 244) discusses the effect of shear lag upon the bending of stiffened plates.

7.2 Elastic Stability

The problem of buckling of stiffened plates is extremely important to the designers of aircraft because of the rigid requirement of aerodynamic smoothness of wing surfaces. For this reason, this area has been investigated by a great number of researchers. Among the more important contributors are the following: Burns and Skogh (Ref. 212), Caldwell (Ref. 214), Dow, Hickman and Rosen (Ref. 228), Gerard (Refs. 234, 235), Gerard and Becker (Ref. 236), Irving and Mullineux (Ref. 245), Klitchieff (Ref. 250), Nowacki (Ref. 262), Pflüger (Ref. 264), Pochop (Ref. 266), Radok (Ref. 267), Rockey and Cook (Refs. 269, 270), Rosenhaupt (Ref. 271), Seide (Ref. 280), Seydel (Ref. 282), Sokolowski (Ref. 284), Strasser (Ref. 285), Timoshenko (Ref. 289), Timoshenko and Gere (Ref. 96), Tsuiji (Ref. 291), Wang and Zuckerberg (Ref. 294), and Yusuff (Ref. 300).

When orthotropic theory applies, the deflections of the stiffened plate are governed by Eq. (127):

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0,$$

and the values of the constants D_x , D_y , and D_{xy} are given by Eqs. (204) or (205) or (206) or can be determined by experimental means, using methods outlined in the next part. The expressions derived in Part 4 can then be used to solve particular problems.

A very high percentage of papers dealing with this area are experimental in nature, and, thus, solutions are generally given in the forms of curves. The extremely large amount of information available renders impossible a summary of basic expressions and data in this report. Since the literature contains data on many different configurations of stiffened panels, it is impossible to present a concise guide here.

For this reason, the analyst is referred to the bibliography of this paper. The titles of most papers adequately describe the specialized topics with which they deal. For excellent summaries, Handbook of Structural Stability, Parts V and VII (Refs. 235 and 236), are particularly recommended. Bruhn's well-known reference (Ref. 122) also contains a great deal of information dealing with the buckling of stiffened panels.

7.3 Summary

The flat plate reinforced by either longitudinal or transverse stringers, or both, remains an extremely important problem in the design of ships, flight vehicles, and bridges. Only the static behavior has been discussed here. However, there are several investigators who have also concerned themselves with the dynamic problem. Among these are: Feldman (Ref. 232), Greenspon (Ref. 23), and McElman, Mikulas and Stein (Ref. 256).

PART 8 - DETERMINATION OF ELASTIC CONSTANTS

In the previous chapters, use is made of orthotropic plate theory to solve various bending and buckling problems. In order that this theory may be applied, it is first necessary to determine the various elastic constants of orthotropy. Let us review the characteristic differential equation for a general loading:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2D_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} + P \\ + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0.$$

In this expression, the shear deformation effects are neglected. Since such a simplification is permissible in many cases, the determination of the three constants, D_x , D_y , and D_{xy} , will first be considered.

In the more refined case, where shear deformations are accounted for (see Paragraph 1.8) three additional constants must be evaluated. The methods of evaluation of all of these values are discussed in this part.

8.1 Bending Stiffness

The original scheme for measuring the bending stiffness of orthotropic plates is given by

Bergsträsser (Ref. 325). The same method is then discussed by Thielemann in 1945, and later by Hearmon and Adams (Ref. 29), Hoppmann et al (Refs. 329-332), and Tsai and Springer (Ref. 338).

First, the relationship between stresses and strains, Eq. (3), is recalled:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{21} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

Since $S_{12} = S_{21}$, there are four elastic constants to be determined in the case where shearing deformations are neglected.

For the determination of the bending constants:

$$S_{11} = \frac{1}{E_x}, \quad S_{22} = \frac{1}{E_y},$$

$$S_{12} = S_{21} = \frac{-\mu_{xy}}{E_y} = \frac{-\mu_{yx}}{E_x},$$

the plate is oriented and supported as shown in Figure 44.

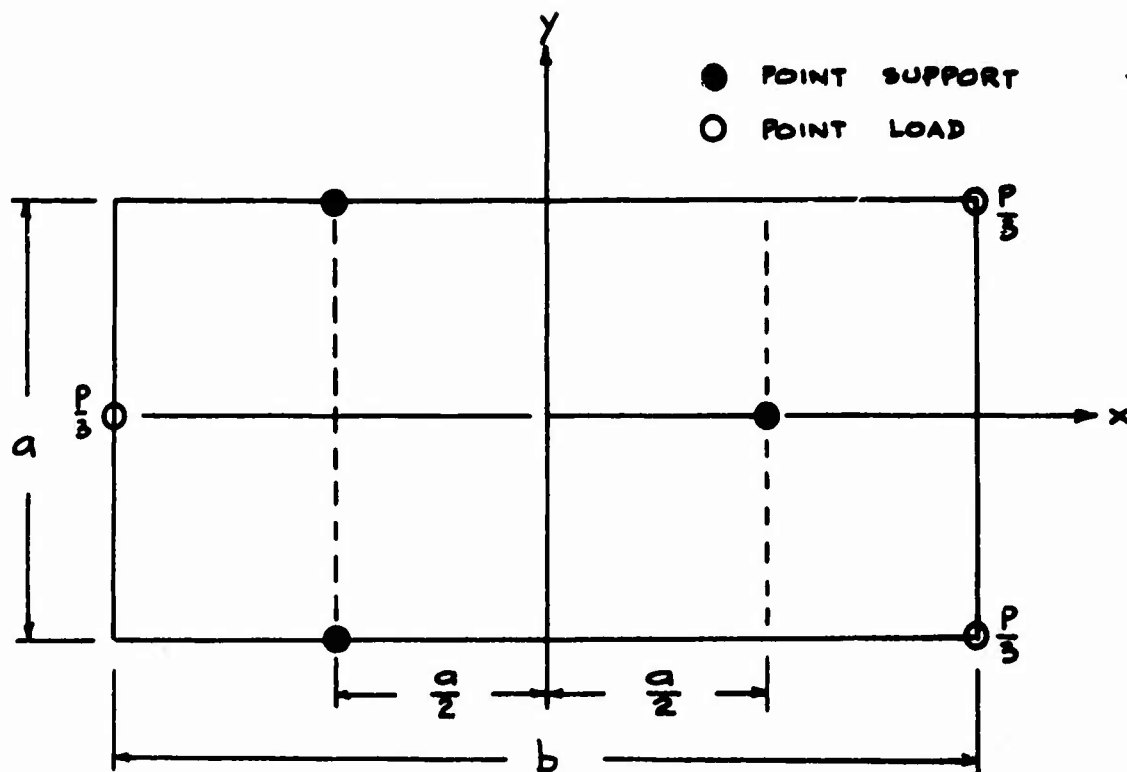


Figure 44
Test for Bending Stiffness (I)

Thus, a uniformly distributed bending moment, M_x , is applied to the plate. Therefore, the deformation at any point (x, y) ,

$$w = \frac{6M_x}{t^3} (s_{11}x^2 + s_{12}y^2) + Ax + By + C, \quad (208)$$

where A, B, and C are constants. The moment in terms of the total applied load, P,

$$M_x = \frac{P}{2a} \left(\frac{b}{2} - \frac{a}{2} \right). \quad (209)$$

From this, the deflection,

$$w = \frac{3P}{4t^3} \left(\frac{b}{2} - \frac{a}{2} \right) (s_{11} x^2 + s_{12} y^2) + A_x + B_y + C. \quad (210)$$

It is now necessary to make two deflection measurements in order to evaluate the constants S_{11} and S_{12} . These two measurements are at

$$x = 0, y = 0,$$

$$x = 0, y = \frac{a}{2}.$$

It is known that the deflection is zero at the supports, given by the coordinates:

$$x = \frac{a}{2}, y = 0,$$

$$x = -\frac{a}{2}, y = \frac{a}{2},$$

$$x = -\frac{a}{2}, y = -\frac{a}{2}.$$

The evaluation of Eq. (210) for these five conditions gives five equations and five unknowns (S_{11} , S_{12} , A , B , C) which are then solved.

The same procedure is followed for the determination of S_{22} and S_{21} , and, thus, the values of the Young's moduli, E_x and E_y , and the Poisson-type constants, M_{xy} and M_{yx} , are known. Thus, the orthotropic stiffnesses, from Eqs. (20) and (21),

$$D_x = \frac{E_x t^3}{12(1 - \mu_{xy} \mu_{yx})} ,$$

$$D_y = \frac{E_y t^3}{12(1 - \mu_{xy} \mu_{yx})} .$$

Another scheme for determining the bending rigidity is presented by Stroud (Ref. 335) and slightly modified by this writer in an unpublished report. The rectangular plate is loaded such that the region between supports is in pure bending (Figure 45).

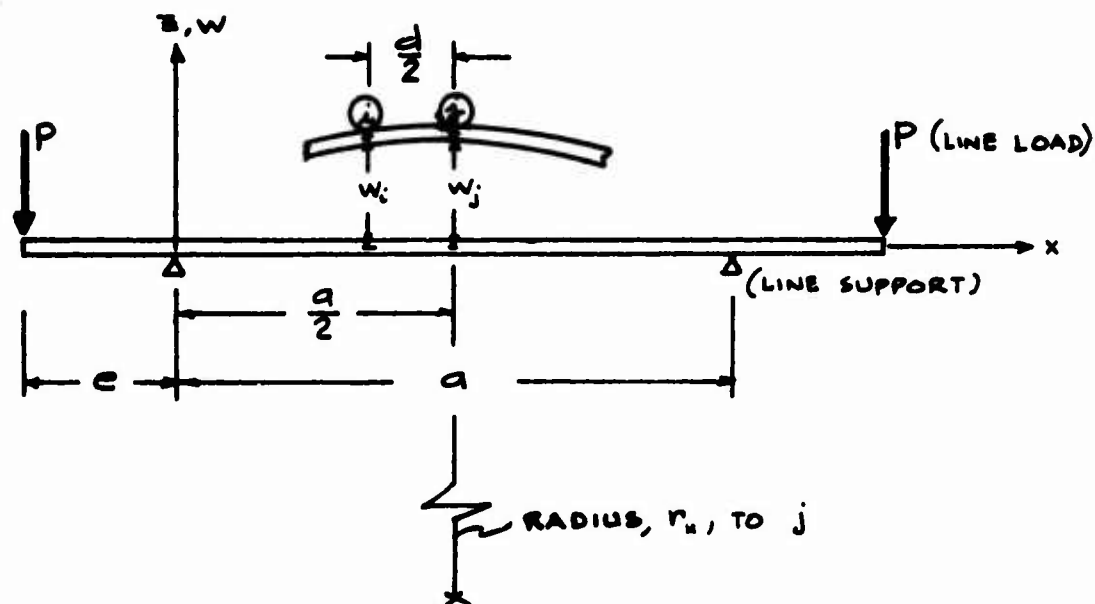


Figure 45
Test for Bending Stiffness (II)

For the shape of the deflected curve,

$$w = kx^2. \quad (211)$$

Differentiating,

$$\frac{\partial^2 w}{\partial x^2} = 2k, \quad \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial x \partial y} = 0.$$

Thus, the bending moment, from Eq. (17),

$$M_x = -2kD_x. \quad (212)$$

The radius of curvature between points i and j is found from the geometry:

$$r = \frac{1}{\left(\frac{\partial^2 w}{\partial x^2}\right)} = \frac{1}{2k} = \frac{4(w_j - w_i)^2 + d^2}{4(w_j - w_i)} \quad (213)$$

Substituting Eq. (213) into Eq. (212), writing the moment in terms of the load, and solving for the stiffness:

$$D_x = \frac{Pe [4(w_j - w_i)^2 + d^2]}{16(w_j - w_i)b} \quad (214)$$

The same procedure is followed for D_y .

8.2 Twisting Stiffness

The loading condition required for the determination of the twist term, D_{xy} , is that of uniformly distributed edge twisting couples. This is accomplished rather simply through the set-up shown in Figure 46, which is described by several writers, among them Timoshenko and Woinowsky-Krieger (Ref. 97), Hoppman et al (Refs. 329-332, 340), Adams (Ref. 29), Stroud (Ref. 335), and Tsai and Springer (Ref. 338).

As is illustrated in Figure 46, a square plate is subjected to concentrated loading at one pair of diagonally opposed corners and supported by point

supports (perhaps ball bearings) at the other two corners.

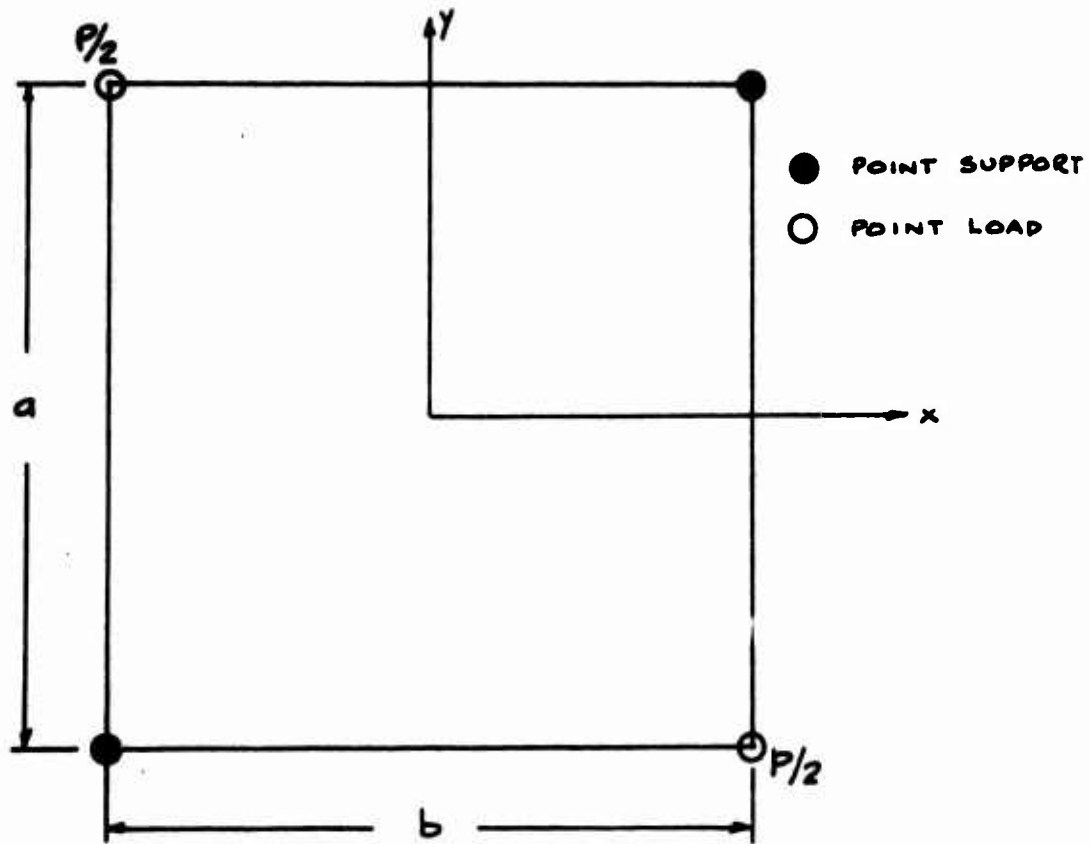


Figure 46
Test for Twisting Stiffness

The theory given by Timoshenko and Woinowsky-Krieger (Ref. 97) shows that the concentrated loads, $P/2$, acting at each of the unsupported corners, produce a torsional moment, M_{xy} , per unit length along each side of

$$M_{xy} = \frac{P}{4} \quad (215)$$

The deformation is given by

$$\left. \begin{aligned} w &= \frac{6M_{xy}}{t^3} s_{66} xy + Ax + By + C \\ &= \frac{3P}{2t^3} s_{66} xy + Ax + By + C. \end{aligned} \right\} (216)$$

Since the supports are at the same height and the deflection at the supports is zero ($w = 0$), the constants A , B , C can be determined. Only one measurement, at

$$x = 0, \quad y = 0$$

is required to solve Eq. (216) for S_{66} . Finally, the relationships,

$$\begin{aligned} s_{66} &= \frac{1}{G_{xy}}, \\ D_{xy} &= \frac{G_{xy} t^3}{6} + \frac{1}{2} (\mu_{xy} D_x + \mu_{yx} D_y), \end{aligned}$$

are used to find the twisting stiffness.

The same testing procedure can be used for a more direct method of calculation. Since the edges of the twisted plate are straight, the plate is in a condition of pure twist with no shears Q_x and Q_y present. Thus, if the deflection, δ , of one of the unsupported corners ($a/2, -a/2$) or ($-a/2, a/2$) is measured, its relationship to the twist is given by the simple relationship,

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{2\delta}{a^2}. \quad (217)$$

From Eq. (19), the torsional rigidity,

$$D_x = \frac{G_{xy} t^3}{6} = \frac{M_{xy}}{\frac{\partial^2 w}{\partial x \partial y}}.$$

From this and Eqs. (217) and (215),

$$D_x = \frac{Pa^2}{8\delta} = \frac{G_{xy} t^3}{6}. \quad (218)$$

This is substituted into

$$D_{xy} = \frac{G_{xy} t^3}{6} + \frac{1}{2} (\mu_{xy} D_x + \mu_{yx} D_y),$$

to get the torsional stiffness.

Great care must be exercised in performing the twist tests. It must be remembered that the edges are to be kept straight. In testing patterns such as corrugations or dimples, care must be taken to allow each cell of the edge cross-section in the xz- and yz- planes to rotate the same amount. The satisfaction of these two requirements may require stiffening of the edges in such a way that the torsional rigidity of the stiffeners is very low while their bending rigidity is high (thin plates), and perhaps filling the edge cells

in such a way that they can be attached to the stiffeners.

Becker and Gerard (Ref. 323) suggest a somewhat different experimental technique in that a cantilevered plate is twisted. But, of course, the twisted plate has the same shape as that discussed above and the method of calculating the constant is the same.

8.3 Shear Stiffness

When the shear deformations of the orthotropic plate are accounted for (Paragraph 1.8), the shear stiffnesses, D_{Q_x} and D_{Q_y} , must be determined. Libove and Batdorf (Ref. 155) and Libove and Hubka (Ref. 156) discuss a method for this calculation.

The transverse shear stiffness, D_{Q_x} , is found by loading the plate as a beam by a uniformly distributed load (Fig. 47).

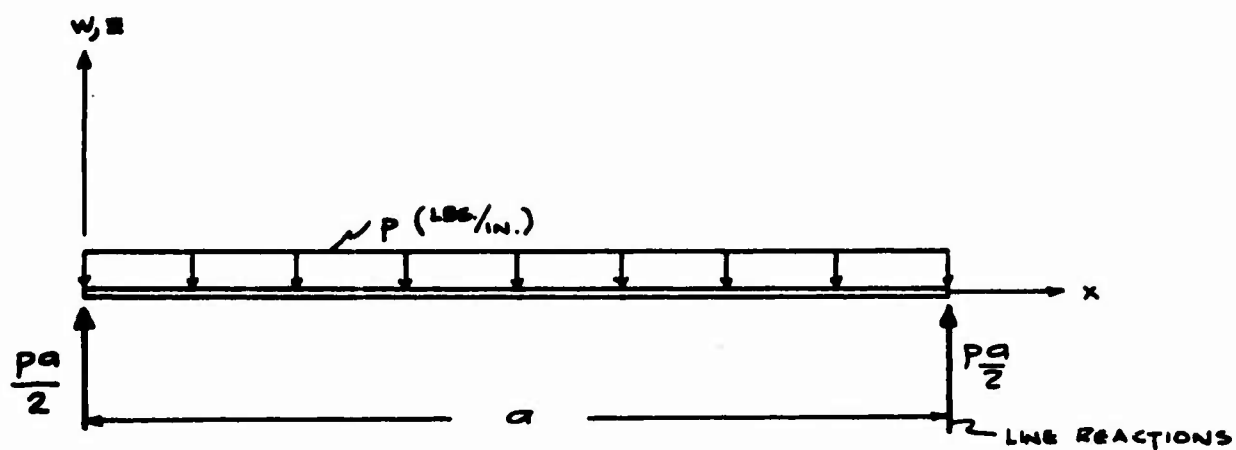


Figure 47
Test for Shear Stiffness

The bending moment at any point,

$$M_x = \frac{1}{b} \left(\frac{pax}{2} - \frac{px^2}{2} \right). \quad (219)$$

The rate of change of transverse shear,

$$\frac{\partial Q_x}{\partial x} = - \frac{p}{b} \quad (220)$$

The curvature, $\partial^2 w / \partial x^2$, is determined from deflection measurements along the x-axis. Then, the shear stiffness is found from:

$$\left. \begin{aligned} D_{ax} &= \frac{\frac{\partial Q_x}{\partial x}}{\frac{\partial^2 w}{\partial x^2} + \frac{M_x}{D_x}} \\ &= - \frac{\frac{P}{b}}{\frac{\partial^2 w}{\partial x^2} + \frac{1}{D_x b} \left(\frac{Pax}{2} - \frac{Px^2}{2} \right)} \end{aligned} \right\} (221)$$

The shear stiffness, D_{ay} , is obviously determined in the same manner.

PART 9 - BIBLIOGRAPHY

A listing of references pertaining to the analysis and design of orthogonally anisotropic plates follows. The references are arranged by subject matter in the following manner:

- Section A: Orthotropic Plate Theory
- Section B: Sandwich Plates
- Section C: Corrugated Plates
- Section D: Externally Stiffened Plates
- Section E: Integrally Stiffened Plates
- Section F: Experimental Determination of
Orthotropic Plate Characteristics
- Section G: Optimization and Synthesis in Design
of Stiffened Plates

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CONCLUSION

The expressions given in this report should enable the analyst to calculate the stresses and deformations of flat plates of various configurations -- sandwich, corrugated, rib-reinforced, integrally stiffened -- due to static loading conditions. Various loading possibilities, in combination with a variety of end support conditions, are described.

Perhaps the greater contribution of this report is the inclusion of the most complete list of references of work in the area of orthotropic plates. The bibliography of Part 9 is divided according to subject matter in such a manner that a researcher in this field will find it convenient in locating publications pertinent to his work.

A second report, which will be more suitable for design, will be published in the near future. In this future publication, the author will present the information concerning orthotropic plates in such a manner that the designer will be able to make rational comparisons between the various configurations of stiffened plates, based upon some standard criterion, such as weight or cost. This work is presently taking place, and it will be presented in a report titled "Orthogonally Stiffened Plates: A Design Handbook."

None

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13. ABSTRACT

✓ A compilation of techniques of analysis for orthogonally stiffened, flat, rectangular plates, with various combinations of loading and support conditions, is presented. The specific types of plates considered are sandwich, corrugated, rib-reinforced, and integrally stiffened. Orthotropic plate theory is used as the major form of solution throughout the report.

A complete bibliography, consisting of more than three hundred references, is included. ↑

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	Honeycomb cores						
	Metal plates						
	Panels (structural)						
	Sandwich panels						
	Sheets						
	Stresses						
	Structural properties						
	Structures						

None
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